ENVIRONMENT MATHEMATICAL ANALYSIS OF FUZZY LOGIC PHENOMENA

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ABSTRACT

In many of the “soft sciences" (e.g., psychology, sociology, ethology) scientists provided verbal descriptions and explanations of various phenomena based on field observations. It is obvious that obtaining a suitable mathematical model, describing the observed system or behavior, can greatly enhance our ability to understand and study it in a scientific manner. The original optimizations mathematical model of the electric arc furnaces charge preheating process mainly takes into consider 2 thermo-technological aspects: the heat transfer between fluids and particles and the heat transfer between the fizz layer and an exchange surface. According the energetically balance at the gaseous environment level, the conductive transfer model is also analyzed through the finished elements method. The outputs of the calculative model are presented as the analysis and quantification of the thermo projects obtained during the charge preheating process. We believe that the fuzzy modeling approach demonstrated here may supply a suitable framework for bio-mimicry, that is, the design of artificial systems based on mimicking a natural behavior observed in nature. These thermo gradients are determined for various temporal moments and for different capacities of the electric arc furnace.

Keywords: **Linguistic modeling, animal and human behavior, territorial behavior, mathematical modeling in biology, hybrid systems, discrete-event systems, bio-mimicry.**

1. INTRODUCTION

Fuzzy logic theory provides the most suitable tool for transforming verbal descriptions into mathematical models. Indeed, the real power of fuzzy logic is in its ability to handle and manipulate verbally-stated information based on perceptions rather than equations (see, e.g., [1, 2, 3, and 4]). Fuzzy modeling is routinely used to transform the knowledge of an expert, be it a physician or a process operator, into a computer algorithm [5, 6]. Yet, not enough attention has been given to its possible use as a tool to assist human observers in transforming their verbal descriptions into mathematical models. A fuzzy model represents the real system in a form that corresponds closely to the way humans perceive it. Thus, the model is understandable, even by non-professionals, and each parameter has a readily perceivable meaning. The model can be easily altered to incorporate new phenomena, and if its behavior is different than expected, it is usually possible to find which rule/term should be modified and how. In addition, fuzzy modeling offers a unique advantage {the close relationship between the verbal description and the resulting mathematical model can be used to verify the validity of the verbal explanations suggested by the observer. Thus, the derived mathematical model can be used to prove or refute the modeler's ideas as to how the natural system behaves and why. The powders are produced during the following technological operations: raw materials loading, steel melting, refining, alloying and evacuation. Generally, the driven powders contain heavy metals (Cr, Ni, Zn, Cd, Pb, Cu etc) and some metal oxides (iron, manganese, aluminum and silicon oxides) and they can reach values of more than 15 kg/t steel [7], [1-3]. Fuzzy Logic (FL) is a powerful problem-solving methodology with wide applications in economical control and information processing. It provides a simple way to draw definite conclusions from vague, ambiguous or imprecise information. It resembles human decision making with its ability to work from approximate data and find precise solutions.

Fuzzy Logic has been found to be very suitable for embedded control applications. Several manufacturers in the automotive industry are using fuzzy technology to improve quality and reduce development time. In aerospace, fuzzy enables very complex real time problems to be tackled using a simple approach. In consumer electronics, fuzzy improves time to market and helps reduce costs. In manufacturing, fuzzy is proven to be invaluable in increasing equipment efficiency and diagnosing malfunctions.

2. ENVIRONMENTAL IMPACT OF EAF

From the total polluting emissions, over 90% are generated during the technological operations of melting and refining. The chemical composition of these emissions is extremely variable and directly dependent on multiple factors, as followings:

- Composition of the raw materials that make up the loading;
- The melting managing way;
- Type of refining process that is used (with gaseous oxygen or ore);
- Time duration of the melting and refining steps;
- Desired quality degree of the elaborated steel.

Figure 1 presents the main scheme of the environment polluting system through the EAF.

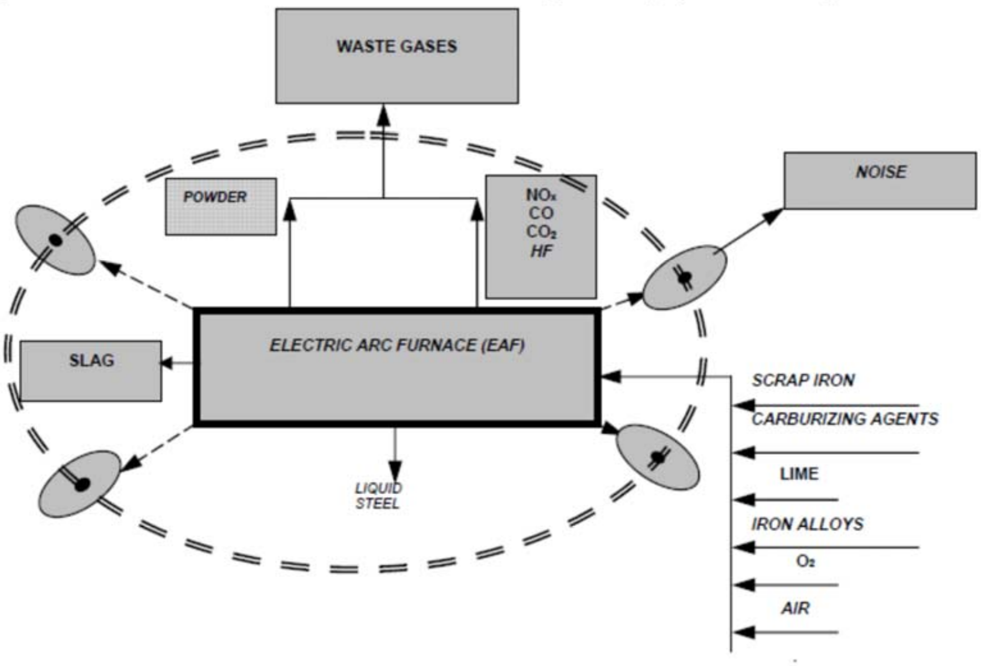

Fig. 1. Environment polluting system through the EA

2.1. Environmental Modeling System of EAF

The modeling system's central element of the EAF processes conceived consists of the system's criteria function. Knowing that the technological processes study for EAF is subordinated to high quality steel obtaining, the modeling system's criteria function (CF) is the ratio between quality and price:

$$CF = \left(\frac{QUALITY}{PRICE} \right)_{\max} \quad (1)$$

The maximum of the criteria function is assured by the mathematical model of prescribing the criteria function (M.P.C.F.) The mathematical model of prescribing the criteria function concept consists of transforming the criteria function (CF) in a quality-economical matrices MQE, as in the scheme presented in fig. 2.

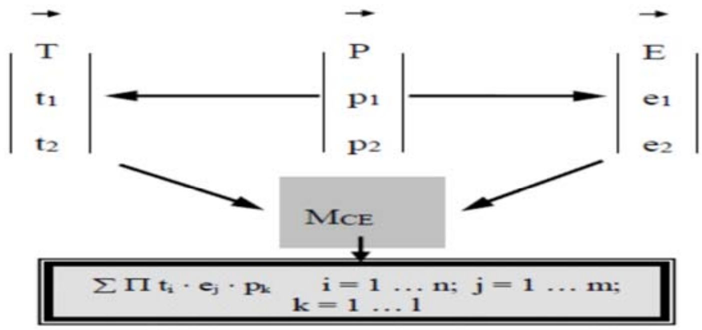

Fig. 2. The modelling system's criteria function's evaluation

The levels of prescribing the criteria function could be obtained by using a composition algorithm for three vectors:

• vector – technical parameters' vector (ti);
• vector – economical parameters' vector (ej);
• vector – weight vector (pk).

2.2. Fuzzy Logic Modelling for Ecological Impact of EAF

Patterns reflect the behavioral characteristics of how a person or a system acts in a certain environment. A spending pattern may represent the way a consumer spends money on different goods, such as travels, cars, or food. A defect pattern in a semiconductor equipment may indicate the way in which a part or assembly fails. By matching various patterns, a marketing specialist at a credit card company is able to better understand consumers spending habits, and therefore he can tailor his or her marketing strategies targeted to different consumer groups (Schneider, 2000). By the same token, scientists study and match various patterns of machine faults in order to be able to predict and control the performance of equipment, including advanced warning.

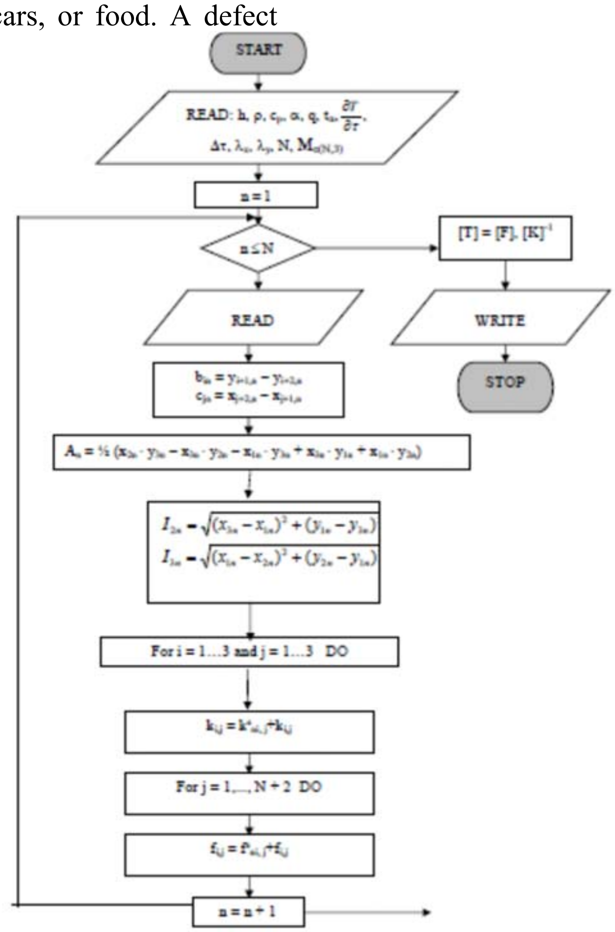

Figure.3. General Logical Scheme

Fuzzy Consumption Utility Functions (FCUF) is based on Utility Theory (UT). Fuzzy Utility Function for Consumption (FUFC) is described by the FUFC matrix concept. This concept is based on the following vectors: S (spending vector); P (price vector); CC (consumption composition vector); C (commodity vector) and SV (service values vector).

3. VERBAL DESCRIPTIONS TO A MATHEMATICAL MODEL

During the 1950s, Forrester and his colleagues developed system dynamics[2] as a method for modeling the dynamic behavior of complex systems. The basic idea is to represent the causation structure of the system using elementary structures that include positive, negative, or combined positive and negative feedback loops. These are depicted graphically, and then transformed into a set of deferential equations. The method was applied successfully to numerous real-world applications in social, economic, and industrial sciences. However, the inherent fuzziness and vagueness of the linguistic description are ignored and exact terms and phrases (e.g., a temperature of 30 Celsius) are modeled in precisely the same way as fuzzy terms (e.g., warm weather).

A more systematic approach to modeling physical systems is qualitative reasoning (QR) which transforms qualitative descriptions of a system into qualitative deferential equations. These are generalizations of deferential equations that include two main components: (1) functional relationships between variables can be represented by functions that are either monotonically increasing or decreasing, but do not have to be completely specified; and (2) the values of the variables are described using a set of landmark values rather than exact numerical values. QR was applied to simulate and analyze many real world systems. However, its applications seem to indicate that it is suitable when modeling with accurate, yet incomplete, knowledge rather than with a verbal description of the system. The starting point in the fuzzy modeling approach is a complete verbal description of the system. This should include the following:

1. The \agents", For example, in a model from the field of ethology describing territorial behavior, the agents might be two animals of the same species.
2. The \environment", that is, the surrounding factors that influence the agents' behavior and interaction. For example, in a model of humans reacting to are alarm this should include the size of the room they are in and the location and size of the exit.
3. The behavior of each agent, that is, its reaction to the other agents and the environment.
4. The overall patterns observed in the natural system as a result of each agent's behavior and the interaction between the agents and their environment.

For example, in a model describing foraging ants, an observed pattern might be that eventually all the ants follow a trail from their nest to the nearest food source. This information is vital because it allows us to validate the mathematical model, once derived. The verbal information is transformed into a mathematical model using the following steps (see the detailed examples in the following sections):

1. Identify the variables. For example, if the model describes an animal that moves in a 3D world, then three variables will be needed to describe the animal's position.
2. Transform the verbal description into fuzzy rules stating the relations between the variables.
3. Define the fuzzy terms (logical operators) in the rules using suitable fuzzy membership functions (mathematical operators).
4. Transform the fuzzy rule base into a mathematical model using fuzzy differencing.

At this point we can analyze and simulate the mathematical model. Its suitability is determined, among other factors, by how well it mimics the patterns that were actually observed in the natural system.

First, it is important that the resulting mathematical model have a simple (as possible) form, so that it will be amenable to analysis. Thus, for example, a Takagi-Sugeno model with singleton consequents might be more suitable than a model based on Zadeh's compositional rule of inference.

Second, when modeling real-worlds systems, the variables are physical quantities with dimensions (e.g., length, time). Dimensional analysis, that is, the process of introducing dimensionless variables, can often simplify the resulting equations and decrease the number of parameters.

3.1. SIMULATIONS

We first simulated the one-dimensional case ($n = 1$) using the parameters:

$$c^1 = -1,\ c^2 = 1,\ a_1 = a_2 = k_1 = k_2 = 1, \tag{2}$$

and initial values $x^1(0) = -0.4$, $x^2(0) = 0.8$, $w_1(0) = w_2(0) = 1$.

Fig. 1 depicts $x^1(t)$ and $x^2(t)$ as a function of t. It may be seen that the fish follow an oscillatory movement with one fish advancing, the other retreating until a point is reached where they switch roles. Finally, they converge to a steady state point at $\overline{x}^1 = -0.1674$ and $\overline{x}^2 = 0.1674$.

We also simulated the three-dimensional case ($n = 3$), this being the one actually found in nature. Fig. 2 depicts the behavior of the model with the parameters: $\mathbf{c}^1 = (-1, -1, -1)^T$, $\mathbf{c}^2 = (1, 1, 1)^T$, $a_1 = a_2 = k_1 = k_2 = 1$, initial values $\mathbf{x}^1(0) = (-0.5, 0.4, -1)^T$, $\mathbf{x}^2(0) = (0.5, 1, 1)^T$, and $w_1(0) = w_2(0) = 1$. It may be seen that again, we have an oscillatory movement converging to a steady state point $\mathbf{x}^i(t) \to \overline{\mathbf{x}}^i$, $i = 1, 2$, with $\overline{\mathbf{x}}^i$ on the line connecting the two nests.

3.2. FUZZY MODELLING

We apply the fuzzy modeling approach to derive a mathematical model of this system. In the first step, we identify the state-variables: fish i, $i = 1, 2$, is located at $\mathbf{x}^i(t) \in \mathbb{R}^n$, and has a *fighting inclination* $w_i(t) \in \mathbb{R}$, and a nest located at $\mathbf{c}^i \in \mathbb{R}^n$.

Next, we transform Lorenz's description of the change in fighting inclination into the following fuzzy rules:

- if $near_i(\mathbf{x}^i, \mathbf{c}^i)$ then $\dot{w}_i = +1$
- if $far_i(\mathbf{x}^i, \mathbf{c}^i)$ then $\dot{w}_i = -1$

that is, the *fighting inclination* increases (decreases) when the fish is near (far) its nest.

Similarly, the description of the movement of fish i is transformed into:

- if $near_i(\mathbf{x}^i, \mathbf{x}^j)$ and $high_i(w_i)$ then $\dot{\mathbf{x}}^i = \mathbf{x}^j - \mathbf{x}^i$
- if $near_i(\mathbf{x}^i, \mathbf{x}^j)$ and $low_i(w_i)$ then $\dot{\mathbf{x}}^i = \mathbf{c}^i - \mathbf{x}^i$

where $\mathbf{x}^j$ is the location of the other fish. That is, when the other fish is near me, and my fighting inclination is high (low), I move in the direction of the other fish (my nest).

The third step is to determine the membership functions for the fuzzy terms. We define $near_i(\mathbf{x}, \mathbf{y})$ using the membership function $n_i(\mathbf{x}, \mathbf{y}) = \exp(-\frac{||\mathbf{x}-\mathbf{y}||^2}{k_i^2})$ (where $k_i > 0$ determines the spread of the Gaussian function). The term $high_i$ is defined using $h_i(w) = \frac{1}{2}(1 + \tanh(\frac{w}{a_i}))$ (where $a_i > 0$ determines the slope of h_i). Note that $\lim_{w \to -\infty} h_i(w) = 0$ and $\lim_{w \to +\infty} h_i(w) = 1$. The opposite terms far_i and low_i are defined using $f_i(x, y) = 1 - n_i(x, y)$, and $l_i(w) = 1 - h_i(w)$, respectively.

$$\begin{aligned} \dot{w}_i &= 2\exp(-\frac{||\mathbf{x}^i - \mathbf{c}^i||^2}{k_i^2}) - 1 \\ \dot{\mathbf{x}}^i &= \mathbf{c}^i - \mathbf{x}^i + h_i(w_i)(\mathbf{x}^j - \mathbf{c}^i) \end{aligned} \tag{1}$$

for $i = 1, 2$. This set of four differential equations constitutes a complete mathematical model of the stickleback behavior described above, and can be used to simulate and analyze the system.

3.3. ANALYSIS

We analyzed the case $n = 1$, that is, the fish live in a 1D world, and the system is described by four state-variables: x^1, x^2, w_1, and w_2. We assume, without loss of generality, that $c^2 = c$ and $c^1 = -c$, for some $c > 0$. In this case, it is easy to verify that (1) admits an equilibrium point:

$$\begin{aligned} \overline{x}^1 &= -c + k_1\sqrt{\log 2}, & \overline{w}_1 &= a_1 \tanh^{-1}\left(2\tfrac{\overline{x}^1+c}{\overline{x}^2+c} - 1\right), \\ \overline{x}^2 &= c - k_2\sqrt{\log 2}, & \overline{w}_2 &= a_2 \tanh^{-1}\left(2\tfrac{\overline{x}^2-c}{\overline{x}^1-c} - 1\right), \end{aligned} \qquad (3)$$

where $\tanh^{-1}$ denotes the inverse hyperbolic tangent function. We assume that the parameters are chosen such that $-c < \overline{x}^1 < \overline{x}^2 < c$ (that is, the equilibrium point is between the two nests, with each fish closer to its nest than the other fish), which implies that the $\overline{w}_i$'s are well-defined.

The next result shows that the arrangement (nest1,fish1,fish2,nest2) is an invariant of the mathematical model.

4. ENVIRONMENTAL IMPACT

The scheme of the EAF environmental impact is presented in fig. 4.

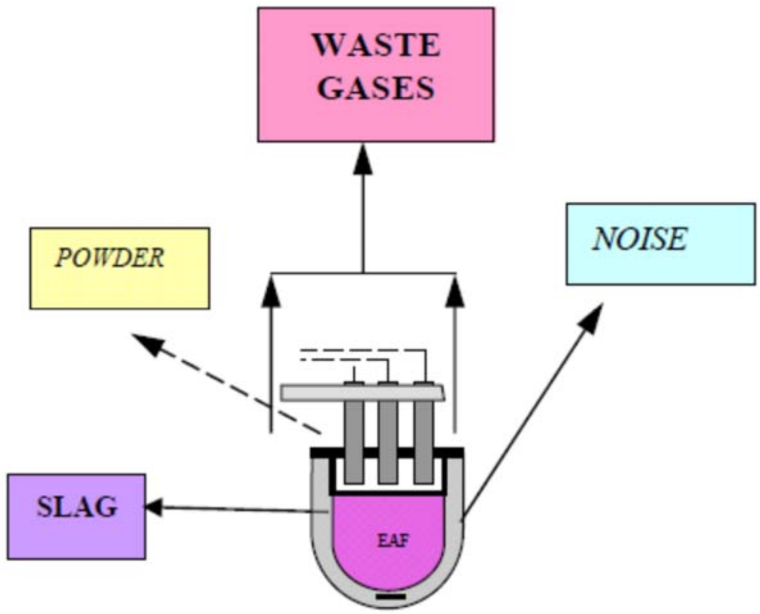

Figure.4.

We can notice the obtaining of:

The criteria function's maximum level FOT,max = 43,76 for the T vector's variation (t1 component - the prescribed variation limits of the elaborated steel quality composition arithmetical mean).

The criteria function's maximum level FOE,max = 55,31 for the E vectors' variation (e3 component - the maximum prescribed productivity of the elaboration process).

And respective the criteria function's maximum level FOCUM,max = 19,85 for the T and E vectors 'cumulated variation.

5. CONCLUSIONS

From the total polluting emissions of EAF, over 90% are generated during the technological operations of melting and refining. The gaseous phase (burnt gases) that comes out of the EAF mainly results from the melting and refining procedures and contains carbon monoxide, carbon dioxide together with nitrogen and sulfur oxides (NOx and SOx); however, in practice it also contains very toxic other components, such as fluorides or volatile organic compounds (dioxine,

chloride derivatives of benzene or phenol) resulted from burning of organic oils that are introduced as contaminants together with the raw materials. Fuzzy Logic has been found to be *very suitable for embedded control applications.* Every application can potentially realize some of the *benefits of FL*: *performance, productivity, simplicity and lower cost. FL algorithms* are implemented as an inference engine which can automatically infer from facts. The application of the fuzzy logic is based on three steps: *fuzzification;* *aggregation* and *defuzzification. Neuro-fuzzy modeling* is to use *neural networks* and *fuzzy set theory* to model practical systems. Neuro-fuzzy technology was developed that can automatically extract business rules from the neuro-fuzzy model. Mathematically, the fuzzy utility function is a more accurate measure on the consumption utility. We believe that fuzzy modeling can and should be utilized in many fields of

science, including biology, economics, psychology, sociology, history and more. In such fields, many verbal models exist in the research literature, and they can be directly transformed into mathematical models using the method described herein.

Recently, the field of bio-mimicry, that is, the design of artificial algorithms and machines that imitate biological behavior, is attracting considerable research interest. In many cases, there exist verbal descriptions of the natural behavior we aim to mimic. Hence, we believe that the paradigm presented here, namely, using fuzzy modeling to transform a behavior described in words into a mathematical model, is particularly suitable for bio-mimicry. Work in this direction is now under way.

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