MODELING, ANALYSIS AND SIMULATION OF ACTIVE MAGNETIC BEARING FOR TWO AXIS POSITION CONTROL OF SHAFT

Cheena Korra
Assistant Professor,Dept. of Electrical & Electronics Engineering
University College of Engineering, Kakatiya University
Kothgudem, Telangana
korrachinna@gmail.com

Abstract

This study deals with the Modeling, Analysis and simulation of an Active Magnetic Bearing for two-axes position control i.e. both the vertical and horizontal directions. First this model is developed for independent nonlinear position control of rotor and is linearised. At the initial Stage the mathematical model is derived for mechanical system in single axis and used for simulation studies in MATLAB. In the next stage the combined effect of electromagnetic model in both the axis is investigated incorporating the mutual effects between the current coils. The mathematical model is analyzed and simulated using MATLAB for nonlinear system and then approximated as a linear control system. The closed-loop response for position control is tuned using the PID Controller. It shows that at stable condition the control current approaches to zero in horizontal axis control. Specifically, we seek the synthesis of control laws with the three attributes as Regulates the rotor position to zero in two axes, Eliminates the steady-state bias flux, and Incorporates the mutual effects between the magnets.

Keywords- ***component; Active Magnetic Bearing; PID Controller; Sencors; PLC;Power Amplifier***

I. INTRODUCTION

Since energy conservation, reliability and high-speed technologies have greater importance, the idea of contact less support of objects with magnetic field becomes more important. Active magnetic bearing System (AMBS) is now being widely used in rotating machinery. Most of the components of AMBS are non-linear [1] therefore the entire system becomes inherently non-linear. The non-linear properties of AMBS can lead to a different behavior as predicted by a linear model. The contact less operation of magnetic bearings offers many advantages e.g. frictionless operation, low vibration, high rotational speed, maintenance-free operation, extend the life of the machinery and usefulness in special environments such as high temperature or vacuum. One attractive alternative that has been investigated in recent years [2] is the magnetic bearing, which uses magnetic forces to suspend a rotor shaft in midair. Magnetic bearings have been used effectively in specialized applications where the higher initial cost can be reduced over time by the performance gains. However, one obstacle to more widespread commercial and industrial application of magnetic bearings is the high sensitivity of the control system to parametric uncertainties and bearing nonlinearities. Methods for feedback control design typically uses a linearized model of the system, but the highly nonlinear properties of the bearing can limit the performance of the overall system. The experimental setup of active magnetic bearing system is shown in figure.1. The feedback control system design technique uses PID controller, Power amplifier, and Programmable Logic Controls (PLC) and these are very effective when the system operation is maintained near the design conditions [3]. The engineers trying to maximize magnetic bearing capabilities are increasingly operating the magnets under conditions in which linear approximate models are not sufficient.

Several works have been done on the non-linear dynamical analysis of the rotor AMB system, and many papers on AMB systems have been published on the bearing design, the bearing locations, the control strategy and the stability analysis. The effects of co-ordinate coupling due to the geometric coupling of the pole arrangement on non-linear behavior are examined in reference [4]. It is important to note that, in most previous work, many authors considered their systems to have been stabilized by feedback control. The actual magnetic force was approximated by a Taylor series expansion about the equilibrium point keeping the lowest-order non-linear terms. The present work examines the behavior of a rotor AMB system in vertical and horizontal directions with critical feedback gains near the double-zero degenerate point. The voltage control strategy is used to take account for the effect of the inductance of bearing magnet. The numerical simulations are also presented to verify the analytical results.

The various applications of magnetic bearings were started to be used in increasing numbers in industrial applications such as turbo molecular pumps, high-speed spindles for machine tools, flywheels for energy storage, reaction wheels for artificial satellites, heavy compressors, turbo generators and jet turbine engines [5]. As a matter of fact, the main function of a bearing is to restrain the motion of the supported structure in response to any applied loads. In active magnetic bearing systems (AMB) the four degrees of freedom of a rotor is controlled by the electromagnets, which were distributed around the periphery of the rotor both for control of x- and y-axis. Since the force between the electromagnets and ferromagnetic rotor is attractive, it is essential to use a closed loop control systems for the stability.

In the literature, two distinct directions is observed, some of the researchers dealt with the design procedure and others concentrated on the control methods. The concept of magnetic suspension was brought up in 1842[4][6]. However in the year 1934 was the real originator of magnetic bearings [4]. Since the main objective for AMBS is developing commercially feasible systems, various kinds of control methods have been used for achieving robust stability. Initial researches on control methods were based on proportional integral-derivative (PID) or proportional derivative (PD) [7] type controllers and reported theoretical results by employing local Proportional-Derivative feedback. The concept of half bridge configuration was used with a PD type controller [8] and reported full state feedback control of a rigid, highly gyroscopic rotor [3]. The H_∞ approach was used for designing a control system [9]. In the article [10] frequency shaped optimal control concept was utilized to improve the unbalance response.

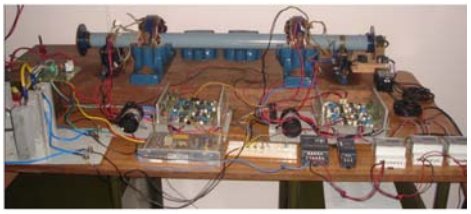

Figure 1. Experimental Setup.

This study commences with developing dynamic equations and modeling of the system. The magnetic force model is linearized and reduced to one linear equation for each axis. The second part of this paper presents the electromagnetic dynamics of active magnetic bearing system. Thus, this model is designed both for x- and y-axes. Since the system is inherently unstable, it is necessary to use a closed loop control system with the feedback of position information of the rotor, for a stable levitation. The third part is devoted to the electromagnetic field analysis and simulation of the system in order to evaluate the effect of the system parameters before the design stage and the simulation results are compared with desired performance values and the final electromagnetic structure is obtained. The final part of this paper is related with the experimental study. The experimental set up is formed which consists of electromagnetic structure, controller, and the position sensors. Fig.2 shows the complete functional block diagram of AMB. The experiments at rest have been conducted for various eccentricities. Finally, it is shown that, this system is capable to produce a stable levitation at standstill. It is expected that, experiments on fully rotating system will be achieved in the near future.

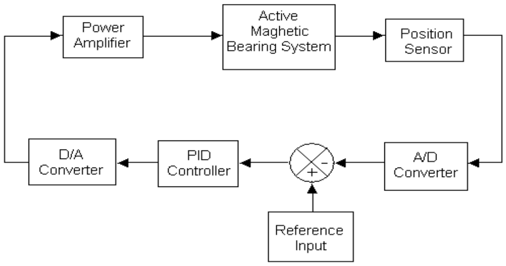

Figure 2. Functional Block Diagram.

This paper is organized as follows. In Section II we present basic concept of system modeling for AMB in vertical and horizontal axis and it is shown that nonlinear relationships between magnetic force, bearing air gap, and magnet currents can be directly canceled by nonlinear state feedback by choosing magnet winding currents as the ‘‘virtual’’ control inputs.

In Section III we present the dynamics of electromagnetic system analysis and to validate the performance of the proposed controller. In Section IV we analysis the design Specifications, hardware characteristics and introduce a new control device ‘Programmable Logic Control’ (PLC) to the nonlinear model. Section V includes various applications and advantages and Section VI we discussed the conclusions with some remarks and directions for further research.

II. SYSTEM MODELLING

Shown in Fig. 1 is a photograph of the disassembled magnetic bearing system used in the experimental work. Figure 3 is a Model of rigid rotor with AMB (A, B) and sensors (C, D, E and F). The sensors C and D are used to indicate the position of shaft in horizontal axis, and sensors E and F indicates the position in vertical axis. The rotor-AMB system shown above is under consideration. A horizontal, uniform symmetric, rigid rotor [11] is suspended by two identical radial AMBS at both ends of the rotor.

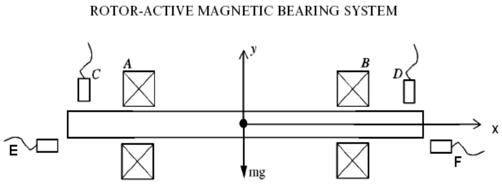

Figure 3. Model of a rigid rotor with AMB (A, B) Sensors(C, D, E and F).

Each bearing is composed of four electromagnets, which are radially set opposite in the horizontal and vertical direction, respectively (Figure 4). In order to simplify the analysis, the magnetic flux leakage, the flux fringing, the eddy current loss [12], and the saturation of the core material between the electromagnets are neglected.

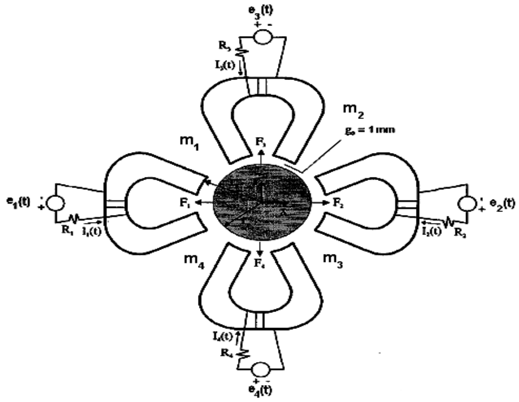

Figure 4. Top of active Magnetic Bearing.

The optical sensor measures vertical and horizontal motion. An aluminum disc has been mounted on the end of the rotor shaft, and sensors are used to detect the vertical and horizontal displacement of the disk. In this work, the dynamics of each axis are considered symmetrical and uncoupled, so a two-axis model is used for analysis and controller design.

III. ELECTROMAGNETIC DYNAMICS

A simplified functional diagram of AMB system is sketched in Fig. 4. Each electromagnet consists of N number of turns of conductor wound around the highly permeable magnetic core. The rotor is modeled as a mass M, and is separated from the magnets by a nominal distance 1mm, hereafter referred to as the ‘‘air gap”.’ Deviation of the rotor from the centered position is denoted by the variable x, which will be referred to as the ‘Position variation’ in horizontal axis and y in vertical axis. The horizontal axis coil currents are denoted by $\mathbf{i}_1$ and $\mathbf{i}_2$, and vertical axis coil currents are denoted by $\mathbf{i}_3$ and $\mathbf{i}_4$. The input voltages at the coil terminals are $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ and $\mathbf{e}_4$. A model for the magnetic flux linkage [1] [4] [14] $\lambda(i, \mathbf{x})$ in each magnet is given by the following equations.

$$\begin{aligned} \lambda_1(i_1, x) &= k\left(\frac{i_1}{1+x}\right) \\ \lambda_2(i_2, x) &= k\left(\frac{i_2}{1-x}\right) \\ \lambda_4(i_4, y) &= k\left(\frac{i_4}{1-y}\right) \\ \lambda_3(i_3, y) &= k\left(\frac{i_3}{1+y}\right) \end{aligned} \qquad (1)$$

Where k is constant. k = (μ_o A_g N^2) / 4, A_g = Area of gap between rotor and magnet, N = Number of turns in each magnet, μ_0= Permeability of material. The inductance of a coil is given by

$$L = N\frac{d\Phi}{di} = \frac{d\lambda}{di} \qquad (2)$$

Therefore the inductance of each coil is given by

$$L_1 = \frac{k}{1+x} \qquad L_2 = \frac{k}{1-x}$$
$$L_3 = \frac{k}{1+y} \qquad (3)$$
$$L_4 = \frac{k}{1-y}$$

When all the four coils are acting simultaneously there will be some mutual effect between the coils. Let the mutual effects between the coils be m_1, m_2, m_3 and m_4. We have the relation for mutual inductance between two inductances L_1 and L_2, i.e. given by

$$m = k'\sqrt{L_1 L_2} \qquad (4)$$

Where k' is the coefficient of coupling and is always less than one. For tight coupling k' is equal to 1. Therefore the magnetic effect between the current coils is given by the following equations.

$$m_1 = \frac{k'k}{\sqrt{(1+x)(1+y)}}$$
$$m_2 = \frac{k'k}{\sqrt{(1-x)(1+y)}}$$
$$m_3 = \frac{k'k}{\sqrt{(1-x)(1-y)}} \qquad (5)$$
$$m_4 = \frac{k'k}{\sqrt{(1+x)(1-y)}}$$

Electromagnetic equations describe the dynamic relationship between electrical variables e & i and magnetic flux λ, while the magneto-mechanical model describes the dynamic relationship between magnetic flux λ and mechanical variables x. Here the bearing system can be modeled in two cases. In first case we can consider the rotor is not in rotation and the second case the rotor is rotating.

A. *Electromagnetic dynamics of Rotor without rotation*

Each axis of the magnetic bearing employs a magnet pair, so the electro -magnetic part of the dynamic model has four nonlinear differential equations of the following form:

$$\begin{aligned}
e_2 &= Ri_2 + L_2\frac{di_2}{dt} + \frac{\partial\lambda_2}{\partial x}\frac{dx}{dt} + m_2\frac{di_3}{dt} + m_3\frac{di_4}{dt} \\
e_1 &= Ri_1 + L_1\frac{di_1}{dt} + \frac{\partial\lambda_1}{\partial x}\frac{dx}{dt} + m_1\frac{di_3}{dt} + m_4\frac{di_4}{dt} \\
e_3 &= Ri_3 + L_3\frac{di_3}{dt} + \frac{\partial\lambda_3}{\partial y}\frac{dy}{dt} + m_1\frac{di_1}{dt} + m_2\frac{di_2}{dt} \qquad (6) \\
e_4 &= Ri_4 + L_4\frac{di_4}{dt} + \frac{\partial\lambda_4}{\partial y}\frac{dy}{dt} + m_3\frac{di_2}{dt} + m_4\frac{di_1}{dt}
\end{aligned}$$

Where **e** is winding input voltage, R is winding resistance; **i** is coil current and λ(i, x) is magnetic flux linkage. First two expressions represent the modeling of AMB system in horizontal direction and the next two expressions represent the modeling of AMB system in vertical direction. Since the opposite coils are connected in parallel, the winding voltages are equal i.e. e_1 = e_2 and e_3 = e_4. In the above equations we can observe that mutual inductance will also affect the rotor gap. The model developed from the above equations are given by

$$\begin{aligned}
\left(\frac{\partial\lambda_2}{\partial x} - \frac{\partial\lambda_1}{\partial x}\right)\frac{dx}{dt} &= R(i_1 - i_2) + L_1\frac{di_1}{dt} - L_2\frac{di_2}{dt} \\
&\quad + (m_1 - m_2)\frac{di_3}{dt} + (m_4 - m_3)\frac{di_4}{dt} \qquad (7) \\
\left(\frac{\partial\lambda_4}{\partial y} - \frac{\partial\lambda_3}{\partial y}\right)\frac{dy}{dt} &= R(i_3 - i_4) + L_3\frac{di_3}{dt} - L_4\frac{di_4}{dt} \\
&\quad + (m_1 - m_4)\frac{di_1}{dt} + (m_2 - m_3)\frac{di_2}{dt}
\end{aligned}$$

Assuming the rotor to be at standstill position and the winding currents of the coils are $i_1 = I_0 + i_{cx}$, $i_2 = I_0 - i_{cx}$, $i_3 = I_a + i_{cy}$ and $i_4 = I_a - i_{cy}$. And x = 0, y = 0

We can observe that Control current in horizontal axis i_{cx} = 0, Mutual inductances between the coils are same. i.e. mutual effect between all the coils are equal, and the inductances of all the coils are equal. The complete behavior of two input two-output system in both vertical and horizontal axis is simulated in next sections.

B. *Electromagnetic Dynamics of Rotor with Rotation*

Consider the rotor is in rotation. When the rotor is rotating, voltage is induced and it will oppose the applied voltage. This induced voltage will effect the rotor position. The voltages due to the rotation of shaft in each coil are given by

$$V_1 = \frac{d}{dt}\left[\frac{ki_1}{1+x}\right]$$
$$V_2 = \frac{d}{dt}\left[\frac{ki_2}{1-x}\right]$$
$$V_3 = \frac{d}{dt}\left[\frac{ki_3}{1+y}\right] \qquad (8)$$
$$V_4 = \frac{d}{dt}\left[\frac{ki_4}{1-y}\right]$$

Now we can develop the electromagnetic model of rotational AMB system. Each axis of the magnetic bearing employs a magnet pair, so the electro -magnetic part of the dynamic model has four nonlinear differential equations of the following form:

$$e_1 = Ri_1 + L_1\frac{di_1}{dt} + \frac{\partial\lambda_1}{\partial x}\frac{dx}{dt} + m_1\frac{di_3}{dt} + m_4\frac{di_4}{dt} + k\frac{d}{dt}\left[\frac{i_1}{1+x}\right]$$
$$e_2 = Ri_2 + L_2\frac{di_2}{dt} + \frac{\partial\lambda_2}{\partial x}\frac{dx}{dt} + m_2\frac{di_3}{dt} + m_3\frac{di_4}{dt} + k\frac{d}{dt}\left[\frac{i_2}{1-x}\right]$$
$$e_3 = Ri_3 + L_3\frac{di_3}{dt} + \frac{\partial\lambda_3}{\partial y}\frac{dy}{dt} - m_1\frac{di_1}{dt} - m_2\frac{di_2}{dt} + k\frac{d}{dt}\left[\frac{i_3}{1+y}\right] \qquad (9)$$
$$e_4 = Ri_4 + L_4\frac{di_4}{dt} + \frac{\partial\lambda_4}{\partial y}\frac{dy}{dt} + m_3\frac{di_2}{dt} + m_4\frac{di_1}{dt} + k\frac{d}{dt}\left[\frac{i_4}{1-y}\right]$$

First two expressions represent the modeling of AMB system in horizontal direction and the next two expressions represent the modeling of AMB system in vertical direction. These equations can be modeled to multi input multi output (MIMO) (two input two-output) system with above assumptions in the first case. The similar analysis with first case can model the above equations as follows

$$\left(\frac{\partial\lambda_2}{\partial x} - \frac{\partial\lambda_1}{\partial x}\right)\frac{dx}{dt} = R(i_1 - i_2) + L_1\frac{di_1}{dt} - L_2\frac{di_2}{dt}$$
$$+(m_1 - m_2)\frac{di_3}{dt} + (m_4 - m_3)\frac{di_4}{dt}$$
$$+k\frac{d}{dt}\left[\frac{i_1}{1+x} - \frac{i_2}{1-x}\right] \qquad (10)$$
$$\left(\frac{\partial\lambda_4}{\partial y} - \frac{\partial\lambda_3}{\partial y}\right)\frac{dy}{dt} = R(i_3 - i_4) + L_3\frac{di_3}{dt} - L_4\frac{di_4}{dt}$$
$$+(m_1 - m_4)\frac{di_1}{dt} + (m_2 - m_3)\frac{di_2}{dt}$$
$$+k\frac{d}{dt}\left[\frac{i_3}{1+y} - \frac{i_4}{1-y}\right]$$

Since the attractive type of electro- magnetic bearing system has an unstable structure, they always require the effective feedback control for the stability. In this study, a PID type controller is chosen to overcome the abrupt changes in the air gap. Although, the system has a nonlinear force-current relationship, a linear controller is used for achieving the stability. Thus, the bearing model was linearized.

IV. SIMULATION MODEL

The second step in the design procedure is developing the simulation model for the AMB. Since, the force interacting between the primary and the secondary of the bearing system is attractive, it is essential to use a closed loop control system for stable levitation. The PID controller is preferred here for controlling the system. Thus, the response of the controller is applied to the electromagnets to bring the rotor into balance. For developing the system simulation model, initially, the PID controller is formed in “simulink” environment and the linear model of AMB is presented as a block diagram (Fig.2) in the closed-loop control system with a feedback, then the model is formed for both axes. The parameters given in Table1 are used for obtaining the simulation results.

Table. 1

M---mass of the shaft	= 25 Kg
μ0---Permeability of Air	= $4\pi*10^{-7}$ H/m
R----resistance of coil	=1 Ω
N----No. of turns of coil	= 270
A_g- Area of pole face	= 336 mm^2
Stable gap b/w magnet & shaft	= 1 mm
g—acceleration due to gravity	= 9.81 m/s^2
Diameter of shaft	= 70 mm
Length of the shaft	= 0.96 m
Inner diameter of bearing	= 72 mm

A. *Simulation Results of Rotor without rotattion*

The simulation of this model with mutual effects is obtained and is given in the following figure.5. The simulation results can be obtained by using the equation (7) with step input. Since all the four coils are acting simultaneously, in this case two PID controllers are used, one for each axis and the following PID values are selected to bring the rotor to standstill position in both directions.

X- axis Controller	Y- axis Controller
K_I = 5.3287	K_I = 5.23
K_P = 0.00001	K_P = 0.00001
K_D = 0.001	K_D = 0.0001

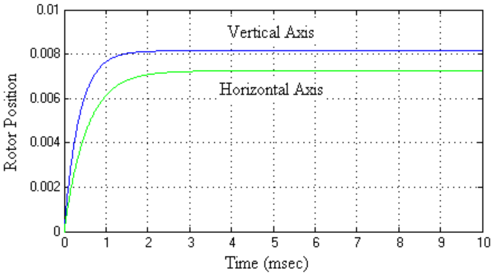

Figure 5. Simulation model of the two axes AMB System.

Fig.5.shows the nonlinear simulation model of the two axes AMB system with rotor is not in rotation. It is a step input responses of the simulation models in x and y directions. The above result shows the position of rotor in *mm* at center of the active magnetic bearing system.

B. *Simulation Results of Rotor with rotattion*

The simulation results of active magnetic bearing system for rotor with rotation when all the four coils acting simultaneously are shown in figure.6. In this analysis two PID Controllers are used, one for each axis and simulation of this model with mutual effects is obtained. The following PID values are selected to bring the rotor to standstill position in both directions.

X-axis Controller	Y-axis Controller
$K_I = 5.392$	$K_I = 5.241$
$K_P = 0.00001$	$K_P = 0.0001$
$K_D = 0.001$	$K_D = 0.0001$

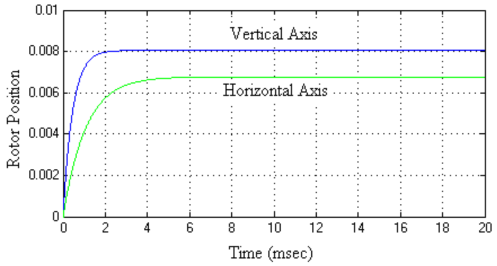

Figure 6. Simulation of AMB system with rotation.

V. EXPERIMENTAL SETUP

The realization of the experimental set up was composed of three stages. Fig.2 shows the experimental set up of AMB. In the first stage the electromagnetic structure and the ferromagnetic rotor have been manufactured with the help of FEM analysis and design optimization. In the second part, since the interaction between the stator and the rotor is inherently unstable, it is essential to use a closed loop control circuit with the feedback of the position information thus, for achieving the best accuracy in the position measurement, a suitable sensor must be chosen. In this study, an optical sensor with analog output was used for the feedback of the rotor position. Input to the Position sensor is the position of the shaft and it gives output as 1 to 5 volts. The controller circuit in the third stage determines the forces need to be applied to the rotor against an eccentricity in the position of the rotor. In the realization stage conception, a PID type controller is used for achieving a stable levitation of the rotor. Thus, PLC (Programmable Logic Controller) is used as an interface between the PC and the hardware of the system, which includes the power electronic circuit. Power amplifier is designed in such a way that it gives 1 to 7 amperes current as output and its input is –10 to +10 volts. Since the force does not depend on the current polarity, a Hybrid configuration is used for increasing and decreasing the current faster. Therefore, in an eccentric position of the rotor the information received by the optical sensors is given as a feedback to the PLC and the PID controller evaluates these received signals. By this closed-loop logic the currents of the upper and lower, right and left electromagnets are defined. After constructing the experimental set up, numbers of experiments have been conducted for various eccentricities. Finally, it is shown that the system is capable to produce almost stable levitation at standstill. Figures.5 and 6 shows the simulation result, which was recorded at standstill.

VI. CONCLUSION

A nonlinear dynamic model for an active magnetic bearing system for two axes position control of the shaft has been presented. This modeling under shaft stationary condition and rotating condition have been evolved. The mutual coupling effect between the coils under two axes operation has been incorporated. Such a method has helped in the controller design to compensate for the mutual interaction between two control loops, namely the x-axis and y-axis in the shaft positioning control system. A stable operation of Active magnetic bearing system is very much needed for artificial heart applications [13]. Simulation of the proposed scheme in MATLAB platform has been carried out. By proper tuning of the controllers a good dynamic performance of the system has been obtained. An objective evaluation of the performance has helped in an appropriate scheme of the PID controller constants.

REFERENCES

[1] John Y. Hung, Nathaniel G.Albritton, Fan Xia, "Nonlinear control of a magnetic bearing system". Auburn University, 200 Broun Hall, Auburn, AL 36849-5201, USA, Mechatronics 13 (2003) 621-637.

[2] L. N. VIRGIN, T. F. WALSH and J. D. KNIGHT, "Nonlinear behavior of a magnetic bearing system". 1995 ASME "Journal of Engineering for Gas and turbines and Power" 117, 582-588.

[3] S. de Queiroz and Darren M. Dawson, "Nonlinear Control of Active Magnetic Bearings: A Backstepping Approach" Marcio Senior Member, IEEE transactions on control systems technology, vol. 4, no. 5, September 1996

[4] A.E. Hartavi, 0. Ustun, R.N. Tuncay, F.Gurleye "The Design, Simulation and Experimental Study of Active Magnetic Bearing". Istanbul Technical University 80626 Maslak-ISTANBULE

[5] N.Motee and M.S.de Queiroz, "Control of Active Magnetic Bearing with a smart bias". Louisiana state university, Baton Rouge, LA 7803-7516, December 2002

[6] Ronald D. Williams, f. Joseph Keith, AND PAUL E. ALLAIRE, "Digital Control of Active Magnetic Bearings" IEEE transactions on industrial electronics, vol. 37, no. 1, February 1990

[7] Sriram Srinivasan and Young Man Cho, "Modeling and System Identification of Active Magnetic Bearing System" United Technologies Research Center 411 Silver Lane, MS 129-15 East Hartford, CT 06108 203-727-2166 ss40utrc.utc.com

[8] Schweitzer, "Active magnetic bearings - chances and limitations" G International Centre for Magnetic Bearings, Zurich, schweitzer@ifr.mavt.ethz.ch

[9] Tingshu Hu, Zongli Lin, Wei Jiang, Paul E. Allaire, "Constrained Control Design for Magnetic Bearing Systems" tingshu@ece.ucsb.edu, zl5y@virginia.edu.

[10] Naohiko Takahashi, Minoru Hiroshima, Haruo Miura, Yasuo Fukushima, "Instability Induced by Iron Losses in Rotor-Active Magnetic Bearing System" Hitachi Industries Co. Ltd., 603, Tsuchiura-shi, Ibaraki 300-0013 Japan.

[11] J. C. J, C. H. HANSEN, "Non-linear oscillations of a rotor in active magnetic bearings". Received 6 December 1999, and in final form 14 June 2000

[12] Ha-Yong Kim and Chong-Won Lee, "Analysis of Eddy-Current Loss for Design of Small Active Magnetic Bearings" IEEE TRANSACTIONS ON MAGNETICS, VOL. 40, NO. 5, SEPTEMBER 2004

[13] J.X.Shen, K.J.Tseng, M.Vilathgamuva, W.K.Chan, "A novel compact PMSM with magnetic bearing for artificial heart applications", IEEE transactions on industry applications", vol.36, no.4, pp.1061-1068, July/August 2000.

[14] B.V.Jayawant,"Electromagnetic levitation and suspension techniques", Edward Arnold (publishers) Ltd., 41 Bedford Square, London.