FUNCTIONALLY GRADED MATERIALS – A REVIEW ON APPLICATION OF FINITE ELEMENT METHOD

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Abstract

This paper focuses in direction of review on the application of Finite Element Methods for determining the effect of variation of temperature and different other parameters for the Functionally Graded Materials to calculate the effect of heat transfer. To analyze the thermal conductivity in Functionally Graded Materials different methods have been investigated. Application of FGMs for steady state heat transfer is analyzed in this work .In FGMs problems are involved having complexity of the shapes and geometry. To overcome these problems FEM in combination with shear deformation theories is reviewed. FEM proved to be effective for solving heat transfer problem of functionally graded materials.

Index Terms— Functionally graded materials (FGMs), Finite element method (FEM), Shear deformation theories.

I. INTRODUCTION

FGMs are those composite materials where the composition or the microstructure is locally varied so that a certain variation of the local material properties is achieved. The main role of the metal constituent in the FGM is to provide the structural support, while the other constituent is to provide heat shielding or thermal barrier when subjected to high temperature environments. Functionally graded materials concept was first developed in 1984 by the group of material scientists in Japan, for application of ultrahigh temperature resistant materials [1-3]. Recently, FGMs have arisen as the new class of materials which reveal smooth variation of material properties mainly across the thickness direction normally; a FGM is made of a combination of ceramic and metal for the purpose of thermal shield against large temperature gradients. Due to its low thermal conductivity, the ceramic material provides the high-temperature resistance whereas the ductile metal integral prevents fracture due to its greater toughness. A huge literature on thermal stress, bending, and vibrations of structures made of functionally graded materials are available. Ceramic-rich surface provides thermal resistance while metal-rich surface provides structural strength and stiffness. FGMs possess a number of advantages, including an improved residual stress distribution, high temperature withstanding ability, higher fracture toughness, improved strength, and reduced stress intensity factors, which make them attractive in many engineering sector applications such as aerospace, aircraft, automobile, defence, biomedical and electronic industries etc. A broad review of processing, design and modelling of FGMs can be found in previous literatures [4, 5]. Numerous numerical and analytical solutions have been presented for the analysis of FGM structures. There are various methods in the literature to analyze the heat conduction in functionally graded materials. In this regard various techniques based on FEM used by different group of FGM scientists are conferred.

II. LITERATURE SURVEY

A. Finite Element Method

Solving the engineering problems by conventional analytical methods is either typical or impossible. In these methods mathematical equations are modelled to define the required variables. For example, stress distribution and displacements in solid components or velocity and pressure in liquid flow might be required. Due to uncertainties involved in such a method, large ‘factor of safety’ or more accurately as factor of ignorance was introduced. With the modern start of high speed electronic digital computers, the prominence in engineering analysis has stirred towards more versatile numerical methods. Finite element method is one of them.

The main rule that involved in finite element method is “DEVIDE and ANALYZE”. The greatest unique feature, which separates finite element method from other methods, is “it divides the given domain into a set of sub domains, called finite ‘elements”. Any geometric shape that allows the computation of the solution or its approximation, or provides necessary relations among the values of the solution at selected points called ‘nodes’ of the sub domain, qualifies as finite element. Division of the domain into elements is called ‘mesh’. Approximate solutions of these finite elements give rise to the solution of the given geometry, which is also an approximate solution.

The approximate solution becomes exact when:

- Division of the given domain into infinite number of sub domains or elements.
- The expression for the primary variable must contain a complete set of polynomials (infinite terms).

Figure 1 shows the flow chart of several FEM methods, which gives a clear outline of FEM evolution.

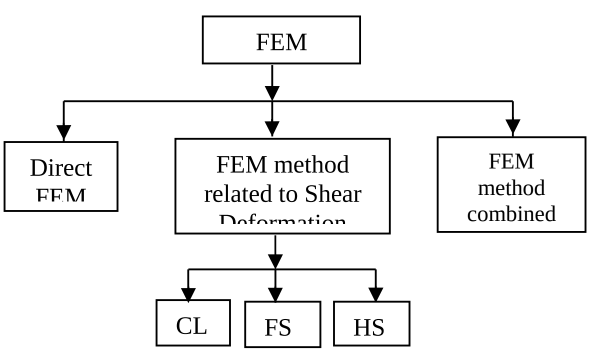

Fig. 1 FEM methods

B. FEM Methods for the Analysis of FGMs

Starting from simple FEM methods, Finite Difference Method, many techniques based on CAD and various other methodologies have been discussed in this section.

Direct Finite Element Method

Simple FEM for the analysis of one dimensional steady state heat conduction problem for composite Elongation Clamped and Bending Free plate under convective heat transfer boundary made up of three layers (ceramic, FGM and metal) is used [1]. Author has formulated the basic equation of i^{th} layer for steady thermal conduction. Principle of minimum potential energy was used to solve the equation for developed element functional [6]. In three layered FGM composite plate the basic steady state heat conduction equation is solved using this functional and FEM based equation is formed i.e. HT = Q where H, Q and T denote thermal stiffness matrix, node thermal load array and unknown node temperature array. The FGM layer thickness effect, FGM layer composition effect and different composite plate’s effect is also studied. Authors have reported smooth curve for FGM layer and in composite layer made of ceramic and metal, there is abrupt change.

Without considering transient effects of temperature change, in this method author shows that in FGM materials properties does not changes abruptly as compared to composites of same materials [1]. Method developed by B. L. Wing considered this point.

Authors have [3] given a solution method for the one-dimensional transient temperature and thermal stress fields for non-homogeneous materials. For space discretization FEM is used, which reduces it in a first-order differential equations systems. Using FEM heat equation can be approximated as:

$$\{C\}\{dT\} + \{K\}\{T\} = \{p\}$$

where, dT denotes differentiation with respect to time, C and K denotes element matrices, p denotes external heat load vector. Using MATLAB code was developed and compared with the exact solutions for some simple cases. For non-homogeneous materials such as FGMs, a finite difference method / finite element method (FDM / FEM) is given to solve the time-dependent temperature field [7]. Finite element space discretization is used to obtain a set of first order differential equations. In high temperature environment application of FGM, one-dimensional transient thermal conduction is of big importance which is solved by present method and results are verified with the solutions present in literature [8–11].

FEM Method Related to Shear Deformation Theories

Composite materials are formed by combining two or more material on a micro scale form and their constituents do not dissolve or merge into each other, to achieve superior enhanced properties. Composite laminates can be treated as plate or shell elements. For the analysis of composite laminates the classical bending theory and shear deformation theories have been employed which are used for plates and shells. For a thin shell having slenderness ratio less than 1/20 the classical bending theory will be adopted. For ratio greater than 1/20, the shear deformation theories will be used. Different authors have given different theories. The oldest and the well-known beam theory is the Euler–Bernoulli beam theory (or classical beam theory—CBT) which assumed that straight lines perpendicular to the mid-plane before bending remain straight and perpendicular after bending. As a result of this assumption, transverse shear strain is neglected. Although this theory is useful for slender beams and plates, it does not give accurate solutions for thick beams and plates. The next theory is the Timoshenko beam theory (the first order shear deformation theory—FSDT) which assumed that straight lines perpendicular to the mid-plane before bending remain straight, but no longer remain perpendicular to the mid-plane after bending. In FSDT, the distribution of the transverse shear stress with respect to the thickness coordinate is assumed constant. The second order shear deformation plate theory further relaxes the kinematic hypothesis by removing the straightness assumption; i.e., the straight normal to the middle plane before deformation may become cubic curves after deformation. The third-order shear deformation theory (TSDT) which assumed parabolic distribution of the transverse shear stress and strain with respect to the thickness coordinate was proposed for beams with rectangular cross-sections (Wang et al, 2000). Also, zero transverse shear stress condition of the upper and lower fibres of the cross-section is satisfied without a shear correction factor in TSDT. The most significant difference between the classical and shear deformation theories is the effect of including transverse shear deformation on the predicted deflections, frequencies, and buckling loads Reddy (2007). But first-order and high-order shear deformation theories are commonly used [22].

FEM Method Related to Classical Layer Plate Theory (CLPT)

This theory is easy and more practical to use with different software packages as compared to FSDT [12]. Using this theory buckling behaviors problem of thin walled box columns was studied. According to classical layer plate theory, FGM layer is assumed to be divided into ‘n’ number of layers, made of different compositions having particular properties in each layer for example in thickness direction thermal conductivity of each layer is different whereas it is assumed constant for each layer and fabrication of these n layers make a FGM layer of desired thickness. The problem is solved by n number of 2-D thin-plate element with 4r nodes per element and 5 degrees of freedom per node. Using the variation approach for the governing equation the following finite element equations [13, 14] are applied to evaluate the temperature along the thickness due to heat conduction:

$K_{cond.}\, T = 0$

where $K_{cond.}$ denotes heat conduction matrix and T denotes the nodal temperature vector. After this, the structural analysis is carried out. For the derivation of FEM equations, the total potential energy principle is used. FEM equations are converted into Eigen value problem. Using the simultaneous iteration methods the buckling Eigen values and buckling mode shapes are calculated.

FEM based on First Order Shear Deformation Theory (FSDT)

The work done by the author [15] is based on First order Shear Deformation Theory (FSDT). Author developed finite element code and formulation for moderately thick rectangular

FGM plates thermal conduction. One dimensional heat conduction equation is used to signify the temperature distribution along thickness of the FGMs plate. Using MATLAB Finite element codes are developed Based on FSDT. Codes developed are validated with results of thermal buckling for isotropic and FGM plates. Developed finite element codes are used to calculate the critical buckling loads for FGM plate having rectangular section with circular cut-out. FGM properties are considered to be temperature dependent. Effects of (i) plate thickness to side ratio, (ii) plate aspect ratio, (iii) size of the cut-out (iv)power index ‘k’ and (v) the boundary conditions on the critical buckling temperature was calculated. In the rectangular FGM plate as radius of cut-out increases critical buckling temperature increases. As thickness to span ratio of FGM plates increases critical buckling temperature again increases. In FGM plates with increase in power law index ‘k’ critical buckling temperature decreases. Critical buckling temperatures of the FGM plate decreases when material properties are considered to be function of temperature in comparison to the results where material properties are assumed to be independent of temperature. Volumetric gradation of FGM is shown in Figure 2. There is 100 % ceramic at top layer and 100 % metal at the bottom, taking thickness of the layer as ‘h’.

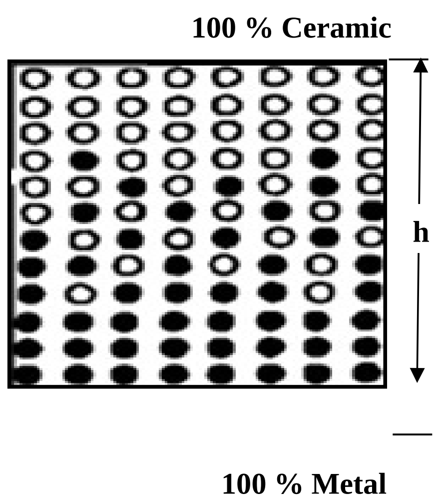

Fig. 2 Functionally graded material

FEM Based on Higher-order Shear Deformation theory (HSDT)

Authors have analyzed the thermal post-buckling problem for a simply supported, composite laminated plate subjected to uniform or non-uniform tent-like temperature loading [16]. Initial geometrical imperfection of plate was considered. Reddy’s Higher-order Shear Deformation Plate Theory including thermal effects is used for problem formulation. A mixed Galerkin-perturbation technique is used to determine thermal buckling loads and post-buckling equilibrium. The effects due to thermal load ratio, transverse shear deformation, total number of plies, plate aspect ratio and fiber orientation are studied. This theory does not require shear correction factors in comparison to FSDT and gives more accuracy. It was concluded that the theory accurately gives transverse shear stresses and displacement compared to previously developed theories for thick plates. Results are very close to three-dimensional elasticity solutions. The effects of material anisotropy, transverse shear deformation, aspect ratio and lamination sequence and fiber orientation on transverse shear stresses are investigated. For a plate of same thickness, as the number of lamina increases, the error in transverse shear stresses decreases. An increase in value of material anisotropy gives lower values of deflection in the plate.

FEM Method combined with CAD

Author [17, 18–20] investigated the material continuity problem and projected different approaches to model complex FGM objects with difficult geometries. Author [21] has given a systematic approach to combine CAD and FEM for FGM materials. A systematic approach to combine these design tools in FGM object design is given in this paper. To design intricate FGM objects, integrated solutions to CAD modelling and property analysis of FGM are used. The material compositions of a particular point are concluded from the CAD models at runtime. Combined FEM of FGM objects are then passed out by establishing a link between the proposed CAD modeller and a commercial FEA package.

With such continuous, complex, and irregular gradations, the design of FGMs object has rarely been studied earlier.

III. CONCLUSIONS

For the FGM Plates analysis of thermal aspects is complex one that's why only limited number of works dealing with thermo-mechanical analysis can be found in literature. The analytical approaches have limitation to certain types of gradation of material properties, simple geometries, and specific type of boundary conditions. Hence numerical methods related to FEM and other methods combined with FEM as discussed in the present work can be used where modelling can be done through CAD software and analysis part with help of FEM. For transient state heat transfer, FEM helps for space discretization and FDM for time dependent response. FEM and related techniques can help for making the complex engineering applications to easier one. It has been validated that the numerical approaches, especially FEM and related techniques are able to handle a wide range of engineering problems, and offer unlimited advantages over conventional numerical methods, specifically for the problems with discontinuities and large deformation. On the other hand, still some challenges are remaining for 3-D modelling of structures. The cost of computation is still too high in case of thin shell structures. For the multi-scale modelling in composite materials and Structures advantages of thermal analysis can be utilized. Now there is no uncertainty that FGMs are showing an important role for providing resistance against thermal fatigue, thermal shock, and temperature resistant materials. Hence FGMs are being used in space shuttles, aircrafts, IC engines, nuclear reactors, turbines, heat exchangers as heat resistant components. Hence research in the field of FGMs is needed of faster pace. For quick and more accurate results, combination of numerical and analytical methods has revealed a great hope. Though much effort has been made for the current area, still there are lot of aspects to be studied.

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