ANALYSIS ON SOME PROBABILITY DISTRIBUTIONS

[1]Anju, [2]Dr. Vineeta Basotia, [3]Dr. Ritikesh Kumar
[1]Research Scholar, Mathematics Department
Shri J.J.T.University, Jhunjhunu Rajasthan
[2]Research Guide, Mathematics Department
Shri J.J.T.University, Jhunjhunu Rajasthan
[3]Research Co. Guide, Mathematics Department
G.C.G. Sec. 14 Gurugram, Haryana

Abstract:
In the present study Probability distributions serve as foundational tools in statistical theory, offering a mathematical framework to model the likelihood of different outcomes in random processes. This abstract explores the overarching concept of probability distributions, delving into their theoretical underpinnings and practical applications. Additionally, it focuses on the application of the generalized gamma distribution in single and double scheme timing tests.

Probability Distribution: Probability distributions are essential in statistics for characterizing the uncertainty associated with random variables. The theoretical foundations of probability distributions encompass concepts like probability density functions, cumulative distribution functions, and moments. Understanding these distributions is crucial for interpreting data and making informed decisions across diverse fields.

Theory and Applications: Theoretical aspects of probability distributions involve exploring their mathematical properties, statistical measures, and the relationships between different distributions. Practical applications extend across disciplines, including finance, biology, and engineering. Probability distributions provide a basis for statistical inference, aiding researchers in drawing meaningful conclusions from observed data and making predictions in uncertain scenarios.

Keywords:Decision-Making,Risk Assessment, Probability Distributions, Everyday Randomness.

Introduction

The term "probability" refers to the likelihood or chance of a particular event occurring. It is a measure quantifying the likelihood of an outcome, typically expressed as a value between 0 (indicating impossibility) and 1 (indicating certainty). In practical terms, probability is used to analyze uncertainty, make predictions, and guide decision-making in various fields such as statistics, mathematics, finance, science, and everyday life. The calculation of probability involves assessing the ratio of favorable outcomes to the total possible outcomes in a given event or situation.

1.1 The basic formula for probability is:

$P(A)$=
Number of Favorable Outcomes/Total Number of Possible Outcomes

This formula expresses the probability $P(A)$ of an event A happening. To use the formula, count the number of outcomes that correspond to the event of interest and divide it by the total number of possible outcomes. The result is a value between 0 and 1, where 0 represents impossibility, 1 represents certainty, and values in between represent different degrees of likelihood.

At its essence, probability quantifies uncertainty by assigning a numerical measure to the likelihood of events occurring. Ranging from 0 (impossible) to 1 (certain), these probabilities provide a framework for reasoning about chance and randomness. The concept is deeply ingrained in our daily lives, influencing choices ranging from weather predictions to financial investments.

1.2 Applications in Decision-Making:

Probability is a linchpin in rational decision-making. Whether in business, finance, or personal choices, understanding the likelihood of different outcomes allows individuals to make informed decisions. From assessing investment risks to predicting the success of a marketing campaign, probability provides a rational basis for navigating the complexities of uncertain scenarios.

Risk Assessment:

In realms such as insurance, healthcare, and engineering, probability plays a pivotal role in risk assessment. Actuaries employ probability models to calculate insurance premiums, healthcare professionals use it to evaluate treatment outcomes, and engineers leverage it to design structures resilient to varying conditions. Probability, in these contexts, acts as a compass for managing and mitigating risks.

Probability Distributions:

Probability distributions formalize the spread of possible outcomes in a random experiment. From the normal distribution's bell curve to the skewed shapes of the gamma distribution, these models offer insights into the patterns of randomness. Statistical analyses, hypothesis testing, and forecasting all lean on the rich tapestry of probability distributions to make sense of observed data.

Everyday Randomness:

Randomness is inherent in everyday events, from coin flips to traffic patterns. Probability helps us comprehend and predict these seemingly unpredictable occurrences. Whether determining the probability of winning the lottery or anticipating the likelihood of a flight delay, probability provides a framework for understanding the inherent variability in life's events.

In conclusion, probability stands as a cornerstone of reasoning in the face of uncertainty. Its applications span a vast spectrum, from guiding decisions to assessing risks and understanding the distribution of random events. As we navigate a world fraught with unpredictability, probability remains an indispensable tool, empowering individuals and societies to make sense of chance and make informed choices. Embracing the principles of probability enhances our ability to decipher the mysteries of randomness, fostering a more informed and resilient approach to the uncertainties that define our lives.

1.3 Introduction to Probability Distributions

Probability distributions are fundamental concepts in the field of statistics and probability theory. They serve as mathematical models for describing the likelihood of various outcomes or events in a random experiment. These distributions play a crucial role in understanding and quantifying uncertainty, making informed decisions, and analyzing data in a wide range of fields, from natural sciences and engineering to social sciences and finance.

In essence, probability distributions provide a structured way to examine and predict the probabilities of different outcomes in a probabilistic system. They can be broadly categorized into two main types: discrete and continuous probability distributions.

Discrete Probability Distributions: Examples of discrete distributions include the Bernoulli distribution, which models binary outcomes (e.g., success/failure), Counting successes events occurring at a steady rate throughout time using the Poisson distribution.

Continuous Probability Distributions: On the other hand, continuous probability distributions apply to random variables that can take any value within a specific range, which is used in many real-world scenarios due to its prevalence in natural phenomena.

In addition to these basic classifications, there are joint distributions, multivariate distributions, and Bayesian distributions, all tailored to more complex statistical problems. Understanding the properties and characteristics of these distributions is essential for various statistical analyses, such as hypothesis testing, parameter estimation, and risk assessment.

Probability distributions also have practical applications in everyday decision-making. For instance, in the world of finance, understanding the probability distribution of investment returns helps investors assess risk and make informed choices. In quality control, they help in identifying defects and anomalies.

This introduction sets the stage for an exploration of the diverse types of probability distributions, their properties, and their applications in statistical analysis, making them indispensable tools in the realm of data analysis and decision science. In the following sections, we will delve into various specific probability distributions, both discrete and continuous, and explore their key characteristics and applications.

What is a Probability Distribution?

An expression that gives the likelihood of various events is called a probability distribution. It specifies how the probabilities are distributed across all possible outcomes. In other words, it describes the pattern of possible results and their associated probabilities.

Here are some key points about probability distributions:

1. **Random Variables:** Probability distributions are linked to random variables. A variable that is dependent on chance in its value is called random or uncertainty. It can take on different values with certain probabilities.
2. **Discrete and Continuous Distributions:** Probability distributions can be classified into two main categories: discrete and continuous.
3. **Probability Mass Function (PMF) and P. d. f.(PDF):** In the case of discrete distributions, the probability distribution is often represented by a PMF, which gives the probability of each possible value. For continuous distributions, the probability distribution is represented by a PDF.
4. **Cumulative Distribution Function (CDF):** The CDF is another important concept related to probability distributions.
5. **Mean, Variance, and Moments:** Probability distributions are characterized by their mean (expected value), variance, and higher moments like skewness and kurtosis. These statistical measures describe and shape of the distribution.
6. **Applications:** Probability distributions are widely used in statistics, data analysis, risk assessment, and decision-making. They help in modeling and analyzing real-world phenomena and making informed predictions. Common applications include finance (e.g., modeling stock returns), biology (e.g., modeling genetic traits), and quality control (e.g., assessing product defects).

Some well-known and uniform distribution, each of which is suited for different types of random events or data.

Understanding and working with probability distributions is a fundamental skill in statistics and data science, as it allows for the quantification of uncertainty and meaningful way.

The Role of Probability Distributions in Statistics

The role of probability distributions in statistics is fundamental and far-reaching. Probability distributions serve as the mathematical underpinning of statistical analysis, providing and random processes. Here are some key aspects of the role of probability distributions in statistics:

1. **Modeling Uncertainty:** Probability distributions are used to model the inherent uncertainty and variability in data and random phenomena. In statistics, we often encounter data that is subject to randomness, and probability distributions offer a way to characterize and make sense of this uncertainty.
2. **Describing Data:** Probability distributions can describe the probability of observing different values in a dataset. They allow statisticians to summarize and analyze data in a meaningful way. For example, many real-world measurements, such as heights or weights.
3. **Parameter Estimation:** Probability distributions often come with parameters that define their shape and characteristics. In statistics, one of the primary tasks is to estimate these parameters from data. For instance, in linear regression, parameters are estimated to describe the relationship between variables.

4. **Hypothesis Testing:** Probability distributions play a crucial role in hypothesis testing. Statisticians use distributions to determine the likelihood of observing a particular outcome under different conditions.
5. **Sampling Distributions:** are used to understand the distribution of sample statistics. Which is a fundamental concept in inferential statistics.
6. **Random Variables:** In statistics, random variables are used to represent uncertain or variable quantities. Probability distributions describe the behavior of these random variables, allowing for the calculation of probabilities associated with different outcomes.
7. **Predictive Modeling:** Probability distributions are central to predictive modeling and machine learning. They enable the quantification of uncertainty in predictions, which is crucial in applications such as forecasting, risk assessment, and classification.
8. **Statistical Inference:** Probability distributions are the basis of statistical inference, which includes tasks like estimating population parameters and testing hypotheses. Inference methods rely on the knowledge of underlying probability distributions.
9. **Quality Control and Process Improvement:** In fields like manufacturing and quality control, probability distributions are used to model and analyze process variations. This helps in identifying defects and improving processes.
10. **Decision Analysis:** Probability distributions are integral to decision analysis and risk assessment. They allow decision-makers to quantify and assess the uncertainty associated with different courses of action.
11. **Simulation:** In simulation studies and Monte Carlo methods, probability distributions are used to generate random inputs and model complex systems. This is valuable in various fields, including finance, engineering, and operations research.

Probability distributions come in a wide variety and many more, each suited for different types of data and applications. Understanding these distributions and their properties is essential for statisticians and data scientists to draw valid conclusions and make informed decisions based on data.

1.4 Discrete Probability Distributions

Discrete probability distributions are used to model random variables that can take on specific, distinct values with known probabilities. These distributions are fundamental in statistics and are commonly employed to analyze and make predictions about discrete data, where the possible outcomes are countable and separate.

1. **Bernoulli Distribution:**
   - The Bernoulli distribution models a binary outcome, where an event can have two possible outcomes, typically labeled as "success" and "failure."
2. **Binomial Distribution:**
   - “The binomial distribution is used to model the number of successful outcomes (successes) in a fixed number of independent Bernoulli trials. It is characterized by two parameters: "n" (the number of trials) and "p" (the probability of success in each trial).”
3. **Poisson Distribution:**
   - “The Poisson distribution models the number of events that occur in a fixed interval of time or space. It is characterized by a single parameter, denoted as "λ" (lambda), which represents the average rate of occurrence.”
4. **Hypergeometric Distribution:**
   - “The hypergeometric distribution is used to model situations involving drawing objects. It describes the probability of drawing a specified number of "successes" in a fixed number of draws. It is characterized by parameters "N" (population size), "K" (number of successes in the population), and "n" (number of draws).”

5. **Geometric Distribution:**
    - “In a series of Bernoulli trials, the geometric distribution represents the number of tries required to obtain the first success. It is defined with just one argument, "p," representing the probability of success in each trial.”
6. **Negative Binomial Distribution:**
    - “By simulating the quantity of trials required to get a certain number of successes, the negative binomial distribution expands upon the geometric distribution. Two factors define it: "r" (number of successes) and "p" (probability of success in each trial)”.
7. **Multinomial Distribution:**
    - “When there are more than two categories or outcomes, may be generalised to create the multinomial distribution. It simulates the quantity of instances in many categories within a predetermined number of trials.”
8. **Discrete Uniform Distribution:**
    - “A finite number of equally plausible possibilities are assigned equal probability by the discrete uniform distribution. A discrete uniform distribution, for instance, can be used to simulate rolling a fair six-sided die.”

These discrete probability distributions are including statistics, engineering, economics, and biology, to analyze and predict outcomes of discrete events. Understanding the properties and characteristics of these distributions is essential for conducting statistical analyses and making informed decisions when dealing with discrete data.

1.5 Continuous Probability Distributions

Continuous probability distributions are mathematical models used to describe the probabilities. Unlike discrete probability distributions, which deal with countable and distinct outcomes, continuous probability distributions apply to situations where outcomes are such as time, distance, or weight. Here are some of the most commonly encountered continuous probability distributions:

1. **Normal Distribution (Gaussian Distribution):**
    - The most well-known continuous probability distribution is probably the normal distribution. It is used to mimic a variety of natural events and has a bell-shaped curve, where data clusters around a central value with a symmetrical spread.
2. **Uniform Distribution:**
    - When modelling scenarios where all values within a certain range have an equal chance of occurring, the uniform distribution is utilised. It distributes in a rectangle form.
3. **Exponential Distribution:**
    - The Poisson process's time between occurrences is modelled using the exponential distribution.
4. **Gamma Distribution:**
    - The gamma distribution is a versatile distribution used to model waiting times, lifetimes, and event counts. It generalizes the exponential distribution.
5. **Weibull Distribution:**
    - It is used in reliability engineering and life data analysis. It models the distribution of failure times.
6. **Log-Normal Distribution:**
    - The log-normal distribution describes data whose logarithms follow a normal distribution. It is commonly used to model data that cannot be negative, such as stock prices.
7. **Cauchy Distribution:**
    - The Cauchy distribution has heavy tails and is often used in physics and engineering, particularly in situations involving resonance and interference.

8. **Uniform Distribution on the Unit Interval:**
    - The uniform distribution, in which any value in the interval [0, 1] has an equal chance, is special to this distribution.
9. **Beta Distribution:**
    - Random variables with a c. p. d. are modeled using the beta distribution. It's frequently applied to Bayesian statistics.
10. **Triangular Distribution:**
    - There is limited information about the underlying distribution, and only minimum, maximum, and most likely values are known.

Each of these continuous probability distributions has its own unique characteristics and applications. They are essential tools in statistics, data analysis, engineering, and various other fields for modeling and understanding continuous data and making informed predictions.

References: -

1. Al-Omari, A.I. and Alsmairan, I.K., (2019), Length-biased Suja distribution and its Applications, Journal of Applied Probability and Statistics, Vol. 14, No. 3, pp. 95-116.
2. Al-Omari, A.I., Al-Nasser, A.D. and Ciavolino, E., (2018), A size-biased Ishita distribution and Application to real data, Quality and Quantity, 53, pp.493-512.
3. Beghriche, A. and Zeghdoudi, H., (2018), A size biased gamma Lindley distribution, Thailand Statistician, 17(2), pp. 179-189.
4. Borah, M. and Hazarika, J., (2017), A new quasi poisson-Lindley distribution: Properties and Applications, Journal of Statistical theory and Applications, Vol. 16,
5. Dar, A.A., Ahmed, A. and Reshi, J.A., (2020), Weighted Gamma-Pareto Distribution and its Application, Pak.J.Statist., Vol. 36(4), pp. 287-304.
6. Eyob, T., Shanker, R., Shukla, K.K. and Leonida, T.A., (2019), Weighted quasi-Akash distribution: Properties and Applications, American Journal of Mathematics and Statistics, 9(1), pp. 30-43.
7. Hassan, A., Wani, S.A. and Shafi, S., (2020), Poisson-Pranav Distribution and its Applications, Pak.J.Statist., Vol. 36(1), pp. 57-72.
8. Mudasir, S. and Ahmad, S.P., (2018), Characterization and estimation of length biased Nakagami distribution, Pak.j.Stat.oper.res., Vol. XIV, No. 3, pp. 697-715.
9. Perveen, Z. and Ahmad, M., (2018), Some properties of size – biased weighted Weibull distribution, International Journal of Advanced and Applied Sciences, 5(5),
10. Rather, A.A. and Subramanian, C., (2018), Length biased Sushila distribution, Universal Review, Vol. 7, Issue 12, pp. 1010-1023.
11. Rather, A.A. and Subramanian, C., (2019), On Weighted Sushila Distribution with Properties and its Applications, International Journal of Scientific Research in Mathematical and Statistical Sciences, Vol. 6, Issue 1, pp. 105-117.
12. Reyad, H.M., Hashish, A.M., Othman, S.A. and Allam, S.A., (2017), The Length biased weighted frechet distribution: Properties and estimation, International Journal of Statistics and Applied Mathematics, 3(1), pp. 189-200.
13. Reyad, H.M., Othman, S.A. and Moussa, A.A., (2017), The Length-Biased Weighted Erlang Distribution, Asian Research Journal of Mathematics, 6(3), pp. 1-15.
14. Shanker, R. and Shukla, K.K., (2017), Power Shanker Distribution and its Application, TurkiyeKlinikleri J Biostat, 9(3), 175-87.
15. Shanker, R. and Shukla, K.K., (2017), Weighted Shanker Distribution and its Applications to Model Lifetime Data, Journal of Applied Quantitative Methods, Vol. 12, No. 2, pp. 1-17.
16. Shanker, R. and Shukla, K.K., (2019), The Poisson-weighted Shanker distribution and its Applications, International Journal of Engineering and

Future Technology, Vol. 16, Issue 3, 70-81.

17. Shanker, R. and Shukla, K.K., (2021), A Zero-truncated Discrete Quasi Lindley Distribution with Applications, TurkiyeKlinikleri J Biostat., 13(1), 1-12.
18. Shanker, R., Shukla, K.K. and Leonida, T.A., (2019), Weighted Quasi Lindley Distribution with Properties and Applications, International Journal of Statistics and Applications, 9(1), 8-20.
19. Shanker, R., Shukla, K.K. and Shanker, R., (2020), A note on size-biased new quasi Poisson-Lindley distribution, Biometrics & Biostatistics International Journal, Vol. 9, Issue 3, 97-104.
20. Shukla, K.K., (2018), Pranav distribution with properties and its applications, Biometrics & Biostatistics International Journal, Vol. 7, Issue 3, 244-254.
21. Shukla, K.K., (2019), Power Pranav Distribution and its Applications to Model Lifetime Data, Journal of Applied Quantitative Methods, Vol. 14, Issue 2.
22. Shukla, K.K., (2020), A Truncated Two Parameter Pranav Distribution and its Applications, Reliability: Theory & Applications, Vol. 15, Issue 3, pp. 103-116.