MARDER'S COSMOLOGICAL MODEL WITH WET DARK FLUID IN SECOND SELF-CREATION THEORY OF GRAVITATION

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Abstract

In this paper we have investigated theMarder's cosmological model with wet dark fluid within the framework of Barber's second self-creation theory. To determine the solution of the field equation we discussed two expansion model as power law model and exponential model and we calculated the physical and kinematical quantities in power law expansion model and exponential expansion model.

Keywords: Barber Second Self-Creation Theory, Deceleration Parameter, Marder's Cosmological Model, Wet Dark Fluid.

1.Introduction

General theory of relativity is a geometric theory of gravitation, providing the description of gravity as a geometric property of space and time. But the recent observation of accelerated expansion of the universe could not be explained by this theory. Nowadays, cosmological model with Dark Energy(DE)in general relatively are widely studied because our universe is undergoing an accelerated expansion. Observation type Ia supernova (SNela) (Riess1998[13], Permutter1999[11]) confirmed currently the universe is expanding with acceleration, from these the universe is flat with undamped form of energy density. This undamped form of energy have negative pressure and is called Dark Energy. There is new candidate for dark energy introduced by Holman and Naidu(2004)[4] called Wet Dark Fluid(WDF). Wet Dark Fluid model is in the spirit of the generalized Chaplygin gas(Gorini 2004). The Wet dark fluid cosmological model for dark energy which derive from an experimental equation of state proposed by Hayward(1967) [2] and Tait(1988)[3] to treat aqueous solutions and water. For Wet dark fluid the equation of state is written as

$$p_{WDF} = \gamma(\rho_{WDF} - \rho^*) \qquad (1)$$

where$p_{WDF}$is pressure of WDF and ρ_{WDF} represents energy density of WDF. In this equationthe parameters $\rho^*$and $\gamma$are taken to be positive and$0 \leq \gamma \leq 1$.

To calculate the energy density of WDF we used conservation equation of energy as

$$\dot{\rho}_{WDF} + 3H(p_{WDF} + \rho_{WDF}) = 0. \qquad (2)$$

By using equation of state of WDF and $3H = \frac{\dot{V}}{V}$, equation (2) becomes

$$\rho_{WDF} = \left(\frac{\gamma}{1+\gamma}\right)\rho^* + \frac{C}{V(1+\gamma)} \qquad (3)$$

where$C$ is constant of integration and $V$is the expansion of volume.

Wet dark fluid includes two components, one component is act as a cosmological constant and another component is act as a standard fluid with $p = \gamma\rho$as a equation of state.If we consider$C > 0$in (3), this fluid will not ignorethe strong energy condition $p + \rho = 0$.Thus we obtained

$$p_{WDF} + \rho_{WDF} = (1+\gamma)\rho_{WDF} - \gamma\rho^*$$
$$= (1+\gamma)\frac{C}{V(1+\gamma)} \geq 0. \qquad (4)$$

The investigation of Wet dark fluid was done by Pawar et al.(2021)[5], in Homogeneous space time with wet dark fluid. Sahoo et al.(2014)[6] have studied Bianchi type VI_1 with wet dark fluidin scale invariant theory of gravitation. S. Singh(2019)[7] evaluated Kaluza-Klein like wet dark fluid in f(R,T) theory of gravitation. Also Mishra et al.(2014)[8] investigated Kaluza-Klein dark energy model in the form of wet dark fluid in f(R,T) gravity.

Barber (1982)[1] introduced two types of self-creation theories usingtwo field equations of general relativity including scalar field and matter. One of this is generalization of Brans-Dicke[17] theory and another one is modification of general relativity. In Barber's second self-creation theory, the gravitational constant of the field equation given by Einstein, is allowed to be a variable scalar on the space-time manifold. In the literature, R. Venkateswarlu et al.(2008)[14]investigated string cosmological solutions in self-creation theory of gravitation. S. Katore et al. (2011)[15] evaluated Einstein-Rosen string cosmological model in Barbers second self-creation theory. A. Nimkar et al. (2021)[16] investigated wet dark fluid cosmological model in Barber self-creation theory of gravitation.

The Marder's space-time has a important role in cosmology and this model plays vital role in understanding the stages of the universe. D. Pawar et al. (2016)[10] derived Marder's space-time in Saez-Ballester theory. S. Aygun (2017)[9] investigated Marder type universe with bulk viscous string cosmological model in f(R,T) gravity. In this paper, we have investigated Marder's space-time with wet dark fluid in Barber's second self-creation theory. This paper contains three sections, in section 2, metric and field equations are presented. In section 3, solutions of Marder's space-time are calculated for both the expansion models and section 4 includes conclusion of the study.

2. Metric and field equations

The Marder's cosmological model is

$$ds^2 = A_1^2(dx^2 - dt^2) + A_2^2 dy^2 + A_3^2 dz^2 \quad (5)$$

Where A_1, A_2, A_3 are the functions of cosmic time t.

The Barber's second self-creation theory field equation is

$$G_{ij} - \frac{1}{2}Rg_{ij} = -8\pi\varphi^{-1}T_{ij} \quad (6)$$

And scalar field equations is

$$\varphi = \varphi^{k}_{;k} = \frac{8\pi\lambda}{3} T \quad (7)$$

whereR_{ij} is Ricci tensor,g_{ij} is Metric tensor, R is scalar curvature,φ is scalar field,the stress-energy tensor of the matter isT_{ij} and T signify its trace.$\lambda$is coupling constant which is calculated from experiments.

Herethe energy momentum tensor for wet dark fluidT_{ij} is

$$T_{ij} = (p_{WDF} + \rho_{WDF})u_i u_j + p_{WDF} \quad (8)$$

Where $p_{WDF}$is pressure of the wet dark fluid, ρ_{WDF} is the density of wet dark fluid, $u^i = (0,0,0,1)$ denotes the four-velocity vector in co-moving co-ordinates and

$$u_i u^i = -1 \quad (9)$$

From (8) and (9) we get

$$T_1^1 = T_2^2 = T_3^3 = p_{WDF} \text{ and } T_4^4 = -\rho_{WDF} \quad (10)$$

From (5), (7), (8) and (10), equation (6) becomes

$$\frac{1}{(A_1)^2}\left(\frac{\ddot{A}_2}{A_2} + \frac{\ddot{A}_3}{A_3} - \frac{\dot{A}_1}{A_1}\frac{\dot{A}_2}{A_2} + \frac{\dot{A}_2}{A_2}\frac{\dot{A}_3}{A_3} - \frac{\dot{A}_1}{A_1}\frac{\dot{A}_3}{A_3}\right) = -8\pi\varphi^{-1}p_{WDF} \quad (11)$$

$$\frac{1}{(A_1)^2}\left(\frac{\ddot{A}_1}{A_1} + \frac{\ddot{A}_3}{A_3} - \frac{(\dot{A}_1)^2}{(A_1)^2}\right) = -8\pi\varphi^{-1}p_{WDF} \quad (12)$$

$$\frac{1}{(A_1)^2}\left(\frac{\ddot{A}_1}{A_1} + \frac{\ddot{A}_2}{A_2} - \frac{(\dot{A}_1)^2}{(A_1)^2}\right) = -8\pi\varphi^{-1}p_{WDF} \quad (13)$$

$$\frac{1}{(A_1)^2}\left(\frac{\dot{A}_1}{A_1}\frac{\dot{A}_2}{A_2} + \frac{\dot{A}_2}{A_2}\frac{\dot{A}_3}{A_3} + \frac{\dot{A}_1}{A_1}\frac{\dot{A}_3}{A_3}\right) = -8\pi\varphi^{-1}\rho_{WDF} \quad (14)$$

$$\ddot{\varphi} + \left(2\frac{\dot{A}_1}{A_1} + \frac{\dot{A}_2}{A_2} + \frac{\dot{A}_3}{A_3}\right)\dot{\varphi} = -\frac{8\pi\lambda {A_1}^2}{3}(3p_{WDF} - \rho_{WDF}) \quad (15)$$

Here the overhead dot ontheA_1, A_2, A_3 denotes ordinary differentiation with respect to time t.

3. Solution of Marder's Space-Time

Comparing equation (12) and (13) we get

$$A_2 = A_3 \quad (16)$$

There are four non-linear differential equations with six unknowns $A_1, A_2, A_3, p_{WDF}, \rho_{WDF}, \varphi$.To determine the solution we have consider some conditions. We consider the condition that the shear scalar σ is in the proportion of the expansion scalar θwhich provide the relation between the metric potential (Collins et al.,1980[18]; Bali, 1986[19]) as

$A_2 = A_1^n$where$n \neq 1$is constant. (17)

Also we consider Hubble's law given by Berman (1983)[22]; Berman and de Mello Gomide (1988)[23] is given by

$$H = \frac{\dot{R}}{R} = BR^{-l} \quad (18)$$

Where$B\ (> 0)$ and $l(\geq 0)$ are constants and R is average scale factor. Here $B \neq 0$ because if $B = 0$ it will results into the static universe and for expanding universe $B > 0$. This law has been given by Singh (2009). The radiation universe is given by

$$3p_{WDF} - \rho_{WDF} = 0 \quad (19)$$

Physical and geometrical parameters are defined as

The deceleration parameter q is defined by

$q = \frac{-R\ddot{R}}{\dot{R}^2}$ (20)

Using equation (18), equation (20) becomes

$q = l - 1$ (21)

Where $l \geq 0$ is constant and therefore we get$q$ is also constant.

The Hubble parameter$H$ is

$H = \frac{1}{3}(H_1 + H_2 + H_3)$ (22)

whereH_1, H_2 andH_3 are the directional Hubble's parameters along X, Y, Zaxis respectively.

$H_1 = \frac{\dot{A}_1}{A_1},\ H_2 = \frac{\dot{A}_2}{A_2},\ H_3 = \frac{\dot{A}_3}{A_3}$ (23)

The average scale factor is denoted by$R$ and is given by

$R = (A_1 A_2^2)^{\frac{1}{3}}$ (24)

The spatial volume V is defined by

$V = R^3 = A_1 A_2^2$ (25)

The Average anisotropic expansion parameter Δ of the universe is

$\Delta = \frac{1}{3}\sum_{i=1}^{3}(\frac{\Delta H_i}{H})^2, \quad \Delta H_i = H_i - H$ (26)

The shear scalar σ^2 is defined by

$\sigma^2 = \frac{1}{2}(\sum_{i=1}^{3} H_i^2 - 3H^2) = \frac{1}{3}(\frac{\dot{A}_1}{A_1} - \frac{\dot{A}_2}{A_2})^2$(27)

The expansion scalar θ is defined by

$\theta = \frac{\dot{A}_1}{A_1} + 2\frac{\dot{A}_2}{A_2}$ (28)

The relation between Hubble's parameter H, the constant l and the age of the universe t is defined by

$t = \frac{H^{-1}}{m}$ (29)

The average scale factor value R is obtained from equation (18) as

$R = (lBt + c_1)^{\frac{1}{l}}$for $l \neq 0$ (30)

and

$R = c_2 e^{Bt}$for $l = 0$ (31)

wherec_1 and c_2 are constants of integration.

3.1 Power law expansion model ($l \neq 0$)

From equation (30) we obtained

$A_1 = (lBt + c_1)^{\frac{3n}{l(n+2)}}$ (32)

$A_2 = A_3 = (lBt + c_1)^{\frac{3}{l(n+2)}}$ (33)

From equation (15) we get, the scalar field as,

$\varphi = \frac{c_5}{(lBt+c_1)^{\frac{6(n+1)+l(n+2)}{l(n+2)}}} + c_4$ (34)

From equation (32) and (33), metric (5) becomes

$ds^2 = (lBt + c_1)^{\frac{6n}{l(n+2)}}(dx^2 - dt^2) + (lBt + c_1)^{\frac{6}{l(n+2)}}(dy^2 + dz^2)$ (35)

For $n \neq 1$, above model (35) represents an anisotropic model.

Physical Quantities of power law expansion model ($l \neq 0$)

The Hubble parameter H is obtained as

$H = B(lBt + c_1)^{-1}$ (36)

The spatial Volume V is given by

$V = R^3 = (lBt + c_1)^{\frac{3}{l}}$ (37)

The average anisotropic expansion parameter Δ is

$\Delta = \frac{2(n-1)^2}{(n+2)^2}, \quad n \neq -2$ (38)

The shear scalar σ^2 is obtained as

$\sigma^2 = \frac{3B^2(n-1)^2}{(n+2)^2(lBt+c_1)^2}, \quad n \neq -2$ (39)

The expansion scalar θ is given by

$\theta = \frac{3B}{(lBt+c_1)}, \quad n \neq -2$ (40)

The value of$q$i.e. deceleration parameter is

$q = l - 1$ (41)

Pressure p_{WDF} of WDF model is

$p_{WDF} = \frac{1}{8\pi}\left[\frac{c_5}{(lBt+c_1)^{\frac{6(n+1)+l(n+2)}{l(n+2)}}} + \right.$

c43B2ln+1n+2−3(n+2)2(lBt+c1)6nln+2 +2, n≠−2 (42)

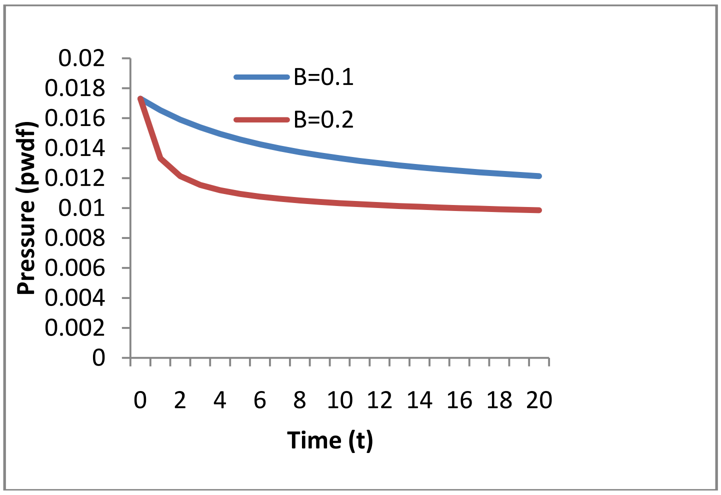

Fig 1.Variation of WDF pressure (p_{WDF}) against time (t) with varying B and choosing appropriate constant.

The graph of WDF pressure against time is decreasing function and as time increases pressure goes to zero (fig.1).

Energy density ρ_{WDF} of WDF model is

$\rho_{WDF} =$

$\frac{3}{8\pi}\left[\frac{c_5}{(lBt+c_1)^{\frac{6(n+1)+l(n+2)}{l(n+2)}}}+\right.$

c43B2ln+1n+2−3(n+2)2(lBt+c1)6nln+2+2, n≠−2 (43)

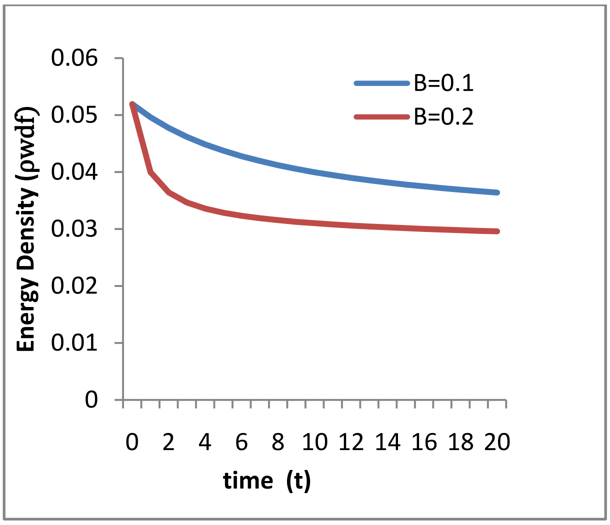

Fig. 2 Variation of WDF energy density (ρ_{WDF}) against time (t)with varying B and choosing appropriate constant.

The graph of WDF energy density against time is decreasing function and as time increases density is zero (fig. 2)

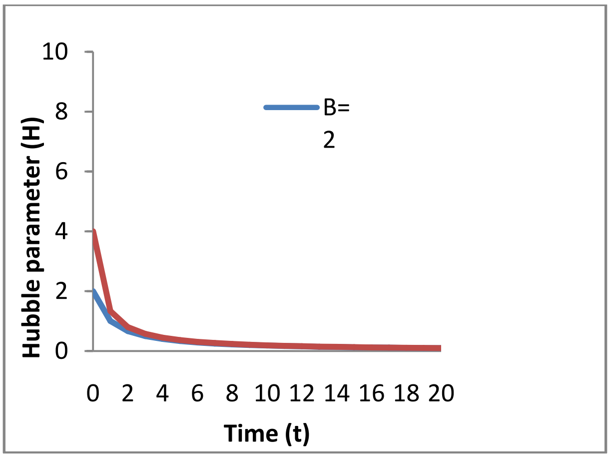

Fig 3.Variation of Hubble parameter (H) against time (t) with varying B and choosing appropriate constant.

The graph of Hubble parameter against time is decreasing function and as time increases function is zero (fig. 3).

3.2 Exponential expansion model ($l = 0$)

From equation (31) we obtained

$$A_1 = c_6^n e^{\frac{3nBt}{(n+2)}} \qquad (44)$$

$$A_2 = A_3 = c_6 e^{\frac{3Bt}{(n+2)}}, \text{ where } c_6 = c_2^{\frac{3}{(n+2)}} (45)$$

From equation (15) we get, the scalar field as

$$\varphi = c_8 e^{3Bt} + c_7, \text{where} c_8 = \frac{1}{3Bc_6^{(n+1)}} \quad (46)$$

In equation (44) and (45) we take $c_6 = 1$ then metric in (5) becomes

$$ds^2 = e^{\frac{6nBt}{(n+2)}}(dx^2 - dt^2) + e^{\frac{6Bt}{(n+2)}}(dy^2 + dz^2)$$
(47)

For $n \neq 1$ above model (47) represents anisotropic model.

Physical Quantities of Exponential Expansion model ($l \neq 0$)

Hubble parameter H is obtained as

$$H = B \text{ where} B > 0 \qquad (48)$$

The spatial Volume V is

$$V = e^{3Bt} \qquad (49)$$

If $B = 0$ then it represent static universe and $B > 0$ represents expanding universe.

The average anisotropic parameter is

$$\Delta = 2\frac{(n-1)^2}{(n+2)^2}, \qquad n \neq -2 (50)$$

The shear scalar $\sigma^2$is

$$\sigma^2 = 3B^2\frac{(n-1)}{(n+2)}, \qquad n \neq -2 (51)$$

The expansion scalar θ is

$$\theta = 3B \qquad (52)$$

The value of$q$ i.e. Deceleration parameter is

$$q = -1 \qquad (53)$$

The pressure p_{WDF} of the WDF is

$$p_{WDF} = -\left(\frac{c_8 e^{3Bt} + c_7}{8\pi}\right)\frac{9B^2}{(n+2)^2 e^{\frac{6nBt}{(n+2)}}}, n \neq -2$$

(54)

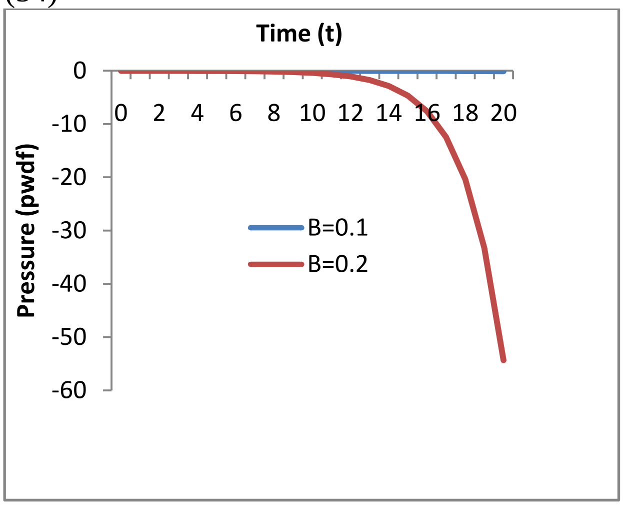

Fig 4.Variation of WDF pressure (p_{WDF}) against time (t) with varying B and choosing appropriate constant.

The graph of WDF pressure against time is increasing function with negative value (fig. 4).

The energy density ρ_{WDF} of the WDF is

$$\rho_{WDF} = -3\left(\frac{c_8 e^{3Bt} + c_7}{8\pi}\right)\frac{9B^2}{(n+2)^2 e^{\frac{6nBt}{(n+2)}}},$$

$n \neq -2$ n (55)

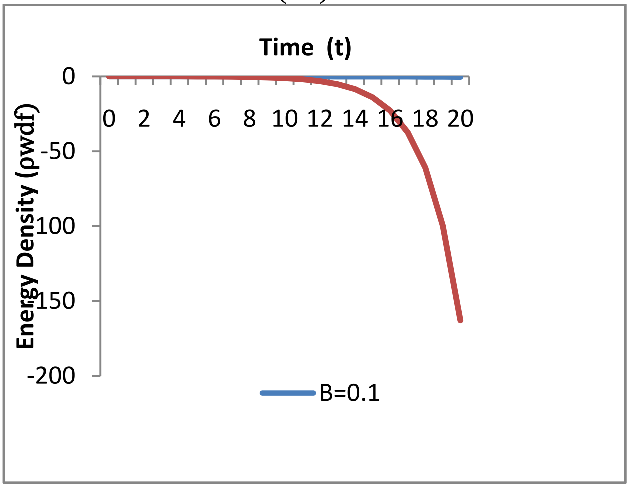

Fig 5 Variation of WDF energy density (ρ_{WDF}) against time (t) with varying B and choosing appropriate constant.

The graph of WDF energy density against time is increasing function with negative value (fig. 5).

4. Conclusion

In the investigation of Marders space-time with wet dark fluid in Self-Creation theory of gravitation, we have obtained the solution of the field equation and we get theconstant value of deceleration parameter in both model i.e.power law expansion model andexponential expansion model. Both the models are anisotropic model and the anisotropic nature of the solution is depends on the value of n.

In power law expansion model,

- The spatial volume vanishes at $t = t_0$ where $t_0 = \frac{-c_1}{lB}$ hence the model has singularity at $t = t_0$ and as $t \to \infty$ the spatial volume increases therefore the model is expanding.
- As time increases the WDF pressure and WDF density approaches to zero.At $t = t_0$, WDF pressure and WDF density are infinite.
- The deceleration parameter q is independent of time t and depends on the value of constant l. If we take $l > 0$ then the value of q is positive which gives decelerated expansion of the universe. If $0 < l < 1$ then the value of q is negative which gives accelerated expansion of the universe.
- The shear scalar and expansion scalar are infinite value at $t = t_0$ and vanished as $t \to \infty$. The Hubble parameter decreases as time increases.
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In exponential expansion model,

- As $t \to \infty$, the spatial volume increases which gives the expanding model.
- The WDF pressure and WDF density are decreases as time increases.
- The value of deceleration parameter is -1 which represents accelerated expansion of the model.
- The shear scalar and expansion scalar does not depends on time t. The Hubble parameter $H = B,\ B > 0$ which represent the rate of expansion increases with increasing the value of B.

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