



# SCFDE SYSTEM USING LINEAR EQUALIZERS FOR MIMO SYSTEM

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**Abstract-** In wireless communication, multipath propagation results in several fading effect. To equalize such long fading channel traditional single carrier time domain equalization becomes infeasible due to its large computational complexity. Single-carrier frequency-domain equalization (SCFDE) offers low complexity, minimum peak to average power ratio as well as less sensitive to carrier frequency offset compared with orthogonal frequency division multiplexing (OFDM). This paper provides SCFDE combining with space time block coding (Alamouti like scheme) for linear equalizers achieves significant diversity gain at low computational complexity over frequency selective fading channel. And compare performance of linear equalizers Zero Forcing and Minimum Mean Square Error. Also here showing that diversity depends not only on antenna configuration and channel memory but also on data block length and data transmission rate.

**Index term-** single carrier, cyclic prefix, zero forcing (ZF) equalization, Minimum Mean Square Error (MMSE) equalization, Alamouti signaling, diversity.

## I. INTRODUCTION

Single carrier frequency domain equalization (SCFDE) was shown to be alternative method for broadband wireless communication instead orthogonal frequency division multiplexing (OFDM). SCFDE avoids several drawbacks of OFDM like high peak to average power ratio (PAPR) and high sensitivity to carrier offset [1].

Also SCFDE provides application for LTE uplink [1], [2].

The MIMO stands for multiple input multiple output. A MIMO wireless communication system has multiple antennas at the transmitter as well as receiver. The very important goal of MIMO wireless technology is to improve quality of communication to improve the bit error rate of signal also data rate of communication. Increase in number of antennas in MIMO system leads to increase in capacity of communication. One of the advantage of MIMO can obtain multiplexing gain and diversity gain linearly increase in the system capacity as well as improving reliability of wireless link. The advantage of MIMO technology can be divided in three categories, spatial multiplexing, transmit diversity technique, and beam forming for enhancing transmission rate, enhancing robustness i.e. BER of transmission using space time coding and other application respectively [3]. Today, there is lot of interest in research on mobile communication system because of increased capacity of MIMO system [4], [5]. An essential problem in wireless communication system is multipath fading, it occurs as signal follows multiple paths between transmit and receive antennas. For certain condition, the incident signals will add up destructively, reducing the received power to zero or near to zero. At this situation no reliable communication is possible. This fading can be mitigated by diversity, which means that the signal is transmitted not only once but several times, it will possible that at least one of the replicas will unaffected by fading.

This paper is analyze the performance of SCFDE combing with Alamouti signaling for linear equalizers. And characterize the diversity as function of transmission block length, channel memory, no of antennas and data rate. The process containing first checking the diversity for different targeted rate. Second obtaining threshold rate (as function of data block length, channel memory, and no of antennas) below which full diversity is achieved.

A brief survey of related literature is as follows. It has been proposed that SCFDE in single antenna (i.e. SISO) system shows a diversity as function of data rate and transmission block length [6]. SCFDE also analyzed for multi-stream MIMO system [7]. Al-Dhahir has proposed Alamouti SCFDE but only shows that the effective channel gain of Alamouti SCFDE is a sum of two independent components and also shows the diversity is two [8]. Design rules are provides for achieving maximum diversity with maximum likelihood (ML) decoding for linear pre-coded OFDM [9]. The linear equalizers achieve maximum multipath diversity in linearly pre-coded OFDM system by Tepedelenlioglu in [10]. The zero padded SC system along with linear equalization was shown that the full diversity is achievable by ZF equalizer [11]. This paper is organized as follows. In section 2 we discuss the basic concept of space-time coding. Section 3 follows brief description on space time block code. Section 4 provides system model for two transmitter receiver antenna system. Section 5 provides the Simulink result and discussion. In section 6 conclusion is given.

## II. SPACE-TIME CODING

Space-time coding (STC), introduced first by Tarokh at el. [12], [13] gives method where the number of the transmitted code symbols per time slot is equal to the number of transmit antennas. Space-time encoder generates the code symbol in manner that the diversity gain, coding gain, as well as high spectral efficiency are achieved. Space-time coding finds its application in cellular communications as well as in wireless local area networks because its property to improve the reliability of data transmission. There are various coding methods as space-time trellis codes (STTC), space-time block codes (STBC), space-time turbo trellis codes and layered space-time (LST) codes. A main issue in all these schemes is the exploitation of redundancy to achieve high

reliability, high spectral efficiency and high performance gain [14]. To design STC first find code matrix which satisfy certain optimality criteria. Also it allows to achieve goals of maintaining a simple decoding algorithm, low error probability and maximizing information rate.

## III. SPACE-TIME BLOCK CODING

The simplest spatial temporal codes is Space-Time Block Codes (STBCs which exploit the diversity for several transmit antennas. [15] Shows Alamouti scheme for simple diversity technique with two transmit antennas gives full diversity. This scheme requires simple linear operation for transmission and reception of data. On block of transmission symbol the encoding decoding process are performed. [14] The transmit diversity technique proposed by Alamouti was space time block code (STBC). By using set of two modulated symbol the encoding decoding operation is performed. Hence, the information data bits are first modulated and mapped into their corresponding constellation points.

Therefore let us assume  $s_0$  and  $s_1$  are two modulated symbols that enters the space-time encoder. Usually for the system with only one transmit antennas, these two modulated symbols are transmitted at two consecutive time instances  $t_1$  and  $t_2$ . The constant time duration  $T$  is the time separation between the  $t_1$  and  $t_2$ . For Alamouti scheme for first time instance the symbol  $s_0$  and  $s_1$  are transmitted by first and second antenna element respectively. Now for second time instance  $t_2$ , the negative of the conjugate of the second symbol, i.e.,  $-s_1^*$  is sent by first antenna at that time conjugate of first constellation point, i.e.,  $s_0^*$  is transmitted by second antenna. The space-time encoding mapping of Alamouti's two-branch -transmit diversity technique can be represented by the coding matrix:

$$S = \begin{bmatrix} s_0 & -s_1^* \\ s_1 & s_0^* \end{bmatrix} \quad (1)$$

The subscript index  $S$  of coding matrix represents the transmit rate. The rows and columns of the matrix represents transmit antennas and different

time instances respectively. For Alamouti system the transmission rate is ‘1’.

IV. SYSTEM MODEL.

A. Alamouti System

The block diagram of Alamouti scheme is shown in fig 1

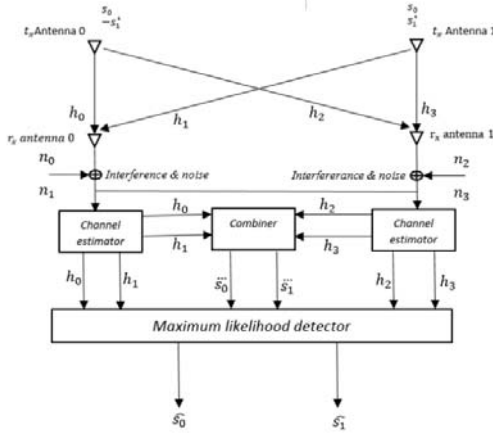


Fig.1. Block diagram of Alamouti's scheme- 2Tx & Rx

For first time slot the received signal is

$$\begin{bmatrix} y_1^1 \\ y_1^1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1 \end{bmatrix} \tag{2}$$

Assume that the channel remains constant for second time slot for transmission of data. The received signal in second time slot is

$$\begin{bmatrix} y_1^2 \\ y_1^2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} -s_1^* \\ s_0^* \end{bmatrix} + \begin{bmatrix} n_2 \\ n_3 \end{bmatrix} \tag{3}$$

Here,

$\begin{bmatrix} y_1^1 \\ y_1^1 \end{bmatrix}$  is represents the received information in time. by received antenna 1,2 respectively.

$\begin{bmatrix} y_1^2 \\ y_1^2 \end{bmatrix}$  is represents the received information in time slot 2 by received antenna 1,2 respectively.

$h_{ij}$  is the channel from  $i^{th}$  received antenna to the  $j^{th}$  transmit antenna.  $s_1, s_2$  are the transmitted symbols.

$\begin{bmatrix} n_0 \\ n_1 \end{bmatrix}$  is represents the noise in time slot 1 for received antennas 1, 2 respectively, and  $\begin{bmatrix} n_2 \\ n_3 \end{bmatrix}$  is represents the noise in time slot 2 for received

antenna 1, 2 respectively. Now combine the equations 2 and 3

$$\begin{bmatrix} y_1^1 \\ y_2^1 \\ y_1^{2*} \\ y_2^{2*} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{12}^* & -h_{11}^* \\ h_{22}^* & -h_{21}^* \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1 \\ n_2^* \\ n_3^* \end{bmatrix} \tag{4}$$

To know the values of  $\begin{bmatrix} s_0 \\ s_1 \end{bmatrix}$ , we have to calculate inverse of H.

Inverse of H:

$$H^+ = (H^H H)^{-1} H^H \tag{5}$$

The term  $(H^H H)$  is,

$$(H^H H) = \begin{bmatrix} h_{11}^* h_{11} + h_{12}^* h_{12} & h_{11}^* h_{21} + h_{12}^* h_{22} \\ h_{21}^* h_{11} + h_{22}^* h_{12} & h_{21}^* h_{21} + h_{22}^* h_{22} \end{bmatrix}$$

(6)

By solving we get,

$$(H^H H) = \begin{bmatrix} |h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2 & 0 \\ 0 & |h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2 \end{bmatrix}$$

(7)

Since this is the diagonal matrix, the inverse is the inverse of diagonal elements, that is

$$(H^H H)^{-1} = \begin{bmatrix} \frac{1}{|h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2} & 0 \\ 0 & \frac{1}{|h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2} \end{bmatrix} \tag{8}$$

Now estimate the transmitted symbols are as follows,

$$\begin{bmatrix} s_0 \\ s_1 \end{bmatrix} = (H^H H)^{-1} H^H \begin{bmatrix} y_1^1 \\ y_2^1 \\ y_1^{2*} \\ y_2^{2*} \end{bmatrix} \tag{9}$$

### B. Linear Equalizer

The linear zero forcing (ZF) and minimum mean square error (MMSE) equalizer is given by [16],

$$\mathbf{W} = (c\rho^{-1} \mathbf{I} + \mathbb{H}^H \mathbb{H})^{-1} \mathbb{H}^H \quad (10)$$

Where the constant  $c = 1$  for MMSE and  $c = 0$  for ZF equalizer. The matrix  $\mathbb{H}$  is the channel which is to be equalized. The definition of diversity gain is,

$$d \triangleq \lim_{\rho \rightarrow \infty} \frac{\log p_e}{\log \rho} \quad (11)$$

and the outage diversity is ,

$$d_{out} \triangleq \lim_{\rho \rightarrow \infty} \frac{\log p_{out}}{\log \rho} \quad (12)$$

Where  $p_e$  is the pairwise error,  $p_{out}$  is the outage probability where  $p_{out}$  is given by

$p_{out} \triangleq \mathbb{P}(I(x; y) < R)$ , here  $I(x; y)$  is mutual information between  $x$  and  $y$ , and  $R$  is target rate.

Now we analyse the ZF equalization for Alamouti transmission scheme. It can be represent as the outage probability of ZF Alamouti STBC scheme is,

$$p_{out} = \mathbb{P}\left(\sum_{k=1}^L \frac{1}{\rho \lambda_k} > \frac{L}{2^{R-1}}\right) \quad (13)$$

The Alamouti scheme of STBC can also be characterized as transmit diversity scheme. We can show that Alamouti signalling preserve the transmit diversity and thus provides larger diversity gain above the threshold rate. Here we consider single carrier block transmission over the frequency selective channel with additive white Gaussian noise with the memory  $\nu$ . The model supports  $2 \times 2$  system and can be extended to  $2 \times N$  system.

Each block of data length  $L$  is appended with a cyclic prefix (CP) of length  $\nu$  to eliminate inter block interference (IBI).  $x_i^{(k)}(n)$  Represents the symbol  $n$  of transmission block  $k$  from antenna  $i$ . At an even time slots, pair of length  $n$  blocks  $x_1^{(k)}(n)$  and  $x_2^{(k)}(n)$  are generated. The transmission scheme is,

$$x_1^{(k)}(n) = -x_2^{*(k)}((-n)_N)$$

$$x_2^{(k)}(n) = -x_1^{*(k)}((-n)_N) \quad (14)$$

for  $n = 0, 1, \dots, N-1$  and  $k = 0, 2, 4, \dots$ ,  $(\cdot)^*$  shows conjugate. CP of length  $\nu$  is added to each transmission block. The total transmission power is divided equally among the antennas.

The transmission scheme is shown in Figure 2.

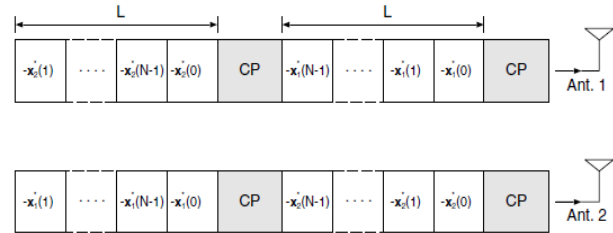


Fig 2. Transmission scheme for communication over frequency selective fading channels.

In time  $k$  and  $k + 1$ , the received signal blocks are given by,

$$y^{(j)} = \sqrt{\rho} \mathbf{H}_1^{(j)} x_1^{(j)} + \sqrt{\rho} \mathbf{H}_2^{(j)} x_2^{(j)} + \mathbf{n}^{(j)}$$

for  $j = k, k + 1$

(15)

Where  $\mathbf{H}_1^{(j)}$  and  $\mathbf{H}_2^{(j)}$  are both circulant matrix, and  $\mathbf{n}^{(j)}$  is noise vector for block  $j$ . A DFT is applied to (15) to diagonalize the channels as follows

$$\mathbf{Y}^{(j)} = \sqrt{\rho} \Lambda_1^{(j)} \mathbf{X}_1^{(j)} + \sqrt{\rho} \Lambda_2^{(j)} \mathbf{X}_2^{(j)} + \mathbf{N}^{(j)}$$

for  $j = k, k + 1$

Where  $\mathbf{Y}^{(j)}$ ,  $\mathbf{X}^{(j)}$  and  $\mathbf{N}^{(j)}$  are the DFT vectors of (15), and  $\Lambda_i$  are diagonal matrix which contain DFT coefficient of channel impulse responses. Assume channel is fixed over two consecutive blocks of  $k$  and  $k + 1$  as follows,

$$\mathbf{Y} = \begin{pmatrix} \mathbf{Y}^{(k)} \\ \mathbf{Y}^{*(k+1)} \end{pmatrix} = \begin{pmatrix} \Lambda_1 & \Lambda \\ \Lambda_2^* & -\Lambda_1^* \end{pmatrix} \begin{pmatrix} \sqrt{\rho} \mathbf{X}_1^{(k)} \\ \sqrt{\rho} \mathbf{X}_2^{*(k)} \end{pmatrix} + \begin{pmatrix} \mathbf{N}^{(k)} \\ \mathbf{N}^{*(k+1)} \end{pmatrix} \quad (16)$$

Now multiplying both the side of (16) by orthogonal matrix  $\Lambda^*$  we get

$$\tilde{\mathbf{Y}} \triangleq \Lambda^* \mathbf{Y} = \begin{pmatrix} \tilde{\Lambda} & 0 \\ 0 & \tilde{\Lambda} \end{pmatrix} \begin{pmatrix} \sqrt{\rho} \mathbf{X}_1^{(k)} \\ \sqrt{\rho} \mathbf{X}_2^{*(k)} \end{pmatrix} + \tilde{\mathbf{N}}$$

(17)

Where  $\tilde{\mathbf{Y}}$  and  $\tilde{\mathbf{N}}$  are the transformed received vector  $\mathbf{Y}$  and noise vector  $\mathbf{N}$  respectively, and  $\tilde{\Lambda}$

$\triangleq \Lambda_1^H \Lambda_1 + \Lambda_2^H \Lambda_2$  is  $N * N$  diagonal matrix whose diagonal element  $i$  is

$$|\Lambda_1(i, i)|^2 + |\Lambda_2(i, i)|^2$$

(18)

### V. SIMULINK RESULT AND DISSCUSION

Fig 1 shows the Alamouti STBC scheme only for two transmit and receive antennas for BPSK modulation over the Rayleigh channel. Transmit diversity scheme Alamouti shows theoretical and Simulink result. Fig 2 shows the Alamouti scheme for zero forcing equalizer with the rate 2, 4, and 6. Fig 3 shows Alamouti scheme with MMSE equalizer for the rate 3, 6, and 9. And the Fig 4 compare Alamouti ZF and MMSE equalizers for the rate 3, 6, and 9.

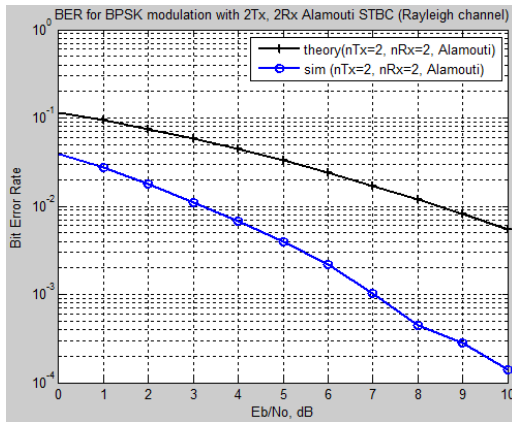


Fig.1. BER for BPSK modulation with 2Tx, 2Rx Alamouti STBC (Rayleigh channel)

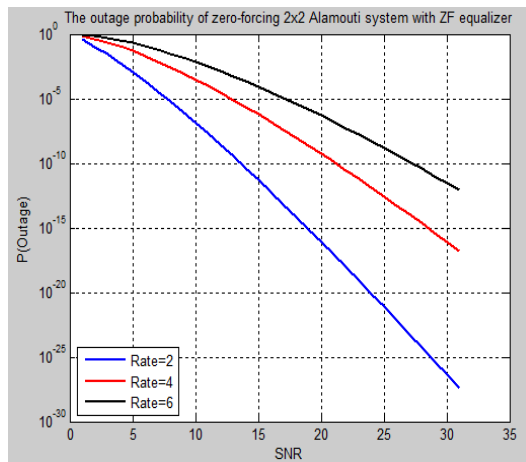


Fig. 2. The outage probability of zero forcing in  $2 \times 2$  Alamouti system for rate 2, 4, and 6.

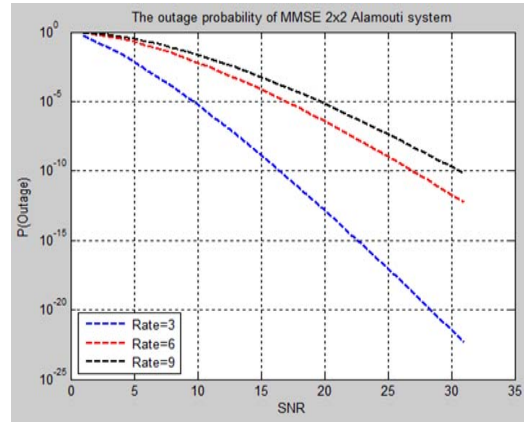


Fig. 3. The outage probability of MMSE in  $2 \times 2$  Alamouti system for rate 3, 6, and 9.

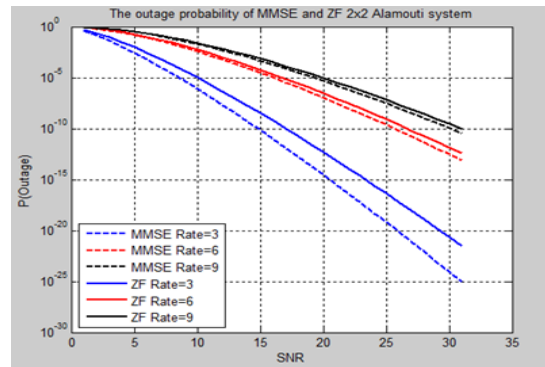


Fig. 4. The outage probability of ZF and MMSE in  $2 \times 2$  Alamouti system for rate 3, 6, and 9.

### VI. CONCLUSION

The transmit diversity scheme Alamouti STBC with  $2 \times 2$  antennas shows bit error rate (BER) decreasing for signal to noise ratio is 10dB. So the Alamouti scheme shows better performance. It is possible because the effective channel information from two receive antennas over two symbols results in diversity order four. In generally can say that  $N_r$  receive antennas, the order of diversity for 2 transmit antenna Alamouti STBC scheme is  $2N_r$ . In calculation  $(H^H H)$  is diagonal matrix represents that there is no cross talk between  $s_0, s_1$  after the equalization and noise term is still white Gaussian. Here characterize the diversity for Alamouti scheme at all spectral efficiency. Which obtain diversity of ZF equalizer is two for all the rate 2, 4, and 6. And the diversity gain of MMSE equalizer is achieves at high rates 3, 6, and 9. So full spatial temporal diversity is achieved. Also shows that diversity depends not only on antenna configuration and channel memory but also on data block length and data transmission rate.

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