



# ANALYSIS AND DESIGN OF STEERING AND SUSPENSION SYSTEM BY MATHEMATICAL AND COMPUTATIONAL METHODOLOGY

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## ABSTRACT

**This paper introduces a design and analysis of suspension and steering system of formula SAE vehicle by both mathematical and computational methodology for optimum performance. The design is according to the formula SAE rulebook. The deep understanding is established between logics and parameters of vehicle. The design parameters are decided either from logics or from worst condition of track and the simulation of parameters are conducted.**

**KEY WORDS: Formula SAE, Suspension Design, Steering Design, ADAMS, Iteration Charts.**

## INTRODUCTION

In Formula SAE International the design is very important due to its regress condition of track the stability and effective handling of vehicles depends upon of designers selection of optimum

steering and suspension geometry which particularly includes the wheel camber, caster and king pin inclination.[1] For light vehicles, advances in modeling techniques are making the analysis of handling behavior a much more realistic process is possible then classical quasi-static techniques.[2] The dependent and independent parameters are decided by mathematical model further the simulation has been done in ADAMS the mass of vehicle, damping coefficient of damper, scrub radius and frequency are independent parameters[3]. Dependent parameters are optimized and calculated by a new and iterative method proposed by this paper, flow charts on vehicle parameter design selection are drafted and tabulated iteration are done. For steering system same methodology is used for calculating the dependent and independent parameter further the test are performed to verify the mathematical model on ADAMS.

## TERMINOLOGY AND DEFINITIONS

S.No	Terminology	Symbol	Unit
1	Mass of vehicle	m	kg
2	Damping coefficient	C <sub>c</sub>	Dimensionless
3	Society of Automotive Engineers	SAE	-
4	Formula SAE	F-SAE	-
5	Kilo metre per hour	kmph	Km/h
6	Wheel centre stiffness coefficient	K	N/m
7	Motion Ratio	M	Dimensionless
8	Damper damping coefficient	C <sub>D</sub>	Ns/m

9	Spring travel	X	mm
10	Longitudinal load transfer	$L_X$	N
11	Longitudinal force according to acceleration	$F_X$	N
12	Height of C.G.	H	mm
13	Wheel base	W	mm
14	Stiffness of spring	$K_S$	N/m
15	Ride frequency	f	Hz
16	Roll gradient	$\theta/a$	deg-s <sup>2</sup> /m
17	Lateral load transfer	$L_Y$	N
18	Distance between roll centre and C.G.	X'	mm
19	Acceleration	a	m/s <sup>2</sup>
20	Torsional rigidity of chassis	$K_t$	Nm/deg
21	Track width	T	mm
22	Lateral Force	$F_Y$	N
23	Rack travel	t	mm
24	Steering arm length	l	mm
25	Caster angle	$\alpha$	deg
26	King pin angle	$\gamma$	deg
27	Steering ratio	$S_R$	Dimensionless
28	Steering angle	$\theta$	deg
29	Scrub radius	$R_S$	mm

## PROBLEM DESCRIPTION

### Independent Parameters

#### Mass

The mass of vehicle is decided by taking care of all the components of vehicle and driver. Special attention is also given to the forces which are generated by the mass, because of this forces condition rise to failure. The decision of mass of vehicle is according to the above constraint.

#### Damper coefficient

Theoretically we have to keep critical damping but due to some losses in the mass-spring damper system; the designer must keep it tending to critical damping but on under damping side. The damping coefficient is taken to die out frequency of system.

#### Scrub radius

It is the distance between centre point of contact patch and point where the steering axis cut the ground, the fundamental reason to take scrub radius is to feel the ride on steering wheel, but it has a problem regarding the large scrub radius

will tends to increase the wear of tyre, hence tyre life get decrease drastically for high scrub radius. Formula type car the ground contact should be strong, the requirement of lateral force at cornering are high thus the optimized scrub radius us needed. One more additional benefit of scrub radius is that at cornering when the wheel base has tendency of change then it will provide more rectangular shape so the stability is maintained.

#### Frequency

The selection of frequency of the system should be such that it should not resonate with any part of human body, otherwise it will result in nervous breakdown of driver. Also the frequency should not match with frequency of the vehicle component also including the air drag experienced by the car.[4]

#### Dependent Parameters

The rest parameters are of dependent type thus table optimization technique is used to decide the parameters

Iteration of Frequency by taking constant motion ratio.

Frequency	f <sub>1</sub> = 4.7	f <sub>2</sub> = 4.8	f <sub>3</sub> = 4.9	f <sub>4</sub> = 5.0	f <sub>5</sub> = 4.6	f <sub>6</sub> = 4.5	f <sub>7</sub> = 4.45
K	52.26x10 <sup>3</sup>	54.50x10 <sup>3</sup>	59.15x10 <sup>3</sup>	59.15x10 <sup>3</sup>	50.06x10 <sup>3</sup>	47.90x10 <sup>3</sup>	46.85x10 <sup>3</sup>
C	37.18x10 <sup>2</sup>	37.97x10 <sup>2</sup>	38.76x10 <sup>2</sup>	39.55x10 <sup>2</sup>	40.34x10 <sup>2</sup>	41.13x10 <sup>2</sup>	23.46x10 <sup>2</sup>
<u>M</u>	<u>0.8</u>	<u>0.8</u>	<u>0.8</u>	<u>0.8</u>	<u>0.8</u>	<u>0.8</u>	<u>0.8</u>
K <sub>s</sub>	81.6 x10 <sup>3</sup>	85.26x10 <sup>3</sup>	92.42x10 <sup>3</sup>	96.46x10 <sup>3</sup>	78.21x10 <sup>3</sup>	74.84x10 <sup>3</sup>	73.2x10 <sup>3</sup>
C <sub>d</sub>	58.09x10 <sup>2</sup>	61.18x10 <sup>2</sup>	67.8x10 <sup>2</sup>	70.37x10 <sup>2</sup>	34.85x10 <sup>2</sup>	34.19x10 <sup>2</sup>	33.51x10 <sup>2</sup>
X	11.022	10.65	9.531	9.14	11.50	12.02	12.5
C <sub>g</sub>	25	23.4	22	21.3	26	28	29

**Iteration Chart: 1**

Iteration of motion ratio by taking frequency constant.

Motion Ratio	M <sub>1</sub> = 0.8	M <sub>1</sub> = 0.85	M <sub>1</sub> = 0.75	M <sub>4</sub> = 0.87
K	46.85x10 <sup>3</sup>	46.85x10 <sup>3</sup>	46.85x10 <sup>3</sup>	46.85x10 <sup>3</sup>
C	23.46x10 <sup>2</sup>	23.46x10 <sup>2</sup>	23.46x10 <sup>2</sup>	23.46x10 <sup>2</sup>
<u>f</u>	<u>4.45</u>	<u>4.45</u>	<u>4.45</u>	<u>4.45</u>
K <sub>s</sub>	73.2x10 <sup>3</sup>	64.84x10 <sup>3</sup>	78.42x10 <sup>3</sup>	61.89x10 <sup>3</sup>
C <sub>d</sub>	33.51x10 <sup>2</sup>	29.8x10 <sup>2</sup>	34.7x10 <sup>2</sup>	26.9x10 <sup>2</sup>
X	12.5	13.88	11.15	14.54
C <sub>g</sub>	29	30	30.8	32

**Iteration Chart: 2**

$$Lx = \frac{Fx \cdot H}{W} \rightarrow X = \frac{Lx}{Ks} \rightarrow Ks = \frac{Lx}{x}$$

$$\downarrow$$

$$F = \frac{1}{2\pi t} \sqrt{\frac{K}{m}} \leftarrow K = Ks \cdot M^2$$

**Flow Chart: 1**

Iteration of C.G. by considering the optimum value of motion ratio as constant.

C.G. height mm	h <sub>1</sub> =180	h <sub>2</sub> =220	h <sub>3</sub> =300	h <sub>4</sub> =362
W	1550	1550	1600	1562.02
L <sub>x</sub>	139.35	170.32	225	278.1
X	14.54	14.54	5	5
K <sub>s</sub>	9.58x10 <sup>3</sup>	11.71x10 <sup>3</sup>	15.47x10 <sup>3</sup>	61.89x10 <sup>3</sup>
F	1.63	1.57	3.78	4.45
<u>M</u>	<u>0.87</u>	<u>0.87</u>	<u>0.87</u>	<u>0.87</u>

**Iteration Chart: 3**

$$\frac{\theta}{a} = \frac{m \cdot X}{Kt} \rightarrow X' = \frac{\theta \cdot Kt}{a \cdot m} \rightarrow Ks = \frac{2 \cdot Kt}{T^2} \rightarrow X = \frac{F}{Ks} \rightarrow K = Ks \cdot M^2 \rightarrow F = \frac{1}{2\pi} \sqrt{\frac{K}{m}}$$

**Flow Chart: 2**

Iterating other parameters by taking the previously determined values as constant.

Track width	T <sub>1</sub> = 1200	T <sub>2</sub> = 1220	T <sub>3</sub> = 1250	T <sub>4</sub> = 1270
h	362	362	362	362
X'	116	195.4	232.76	231.032
K <sub>s</sub>	61.89x10 <sup>3</sup>	61.89x10 <sup>3</sup>	61.89x10 <sup>3</sup>	61.89x10 <sup>3</sup>
K <sub>t</sub>	44.56x10 <sup>3</sup>	46.9x10 <sup>3</sup>	48.34x10 <sup>3</sup>	49.9x10 <sup>3</sup>
<u>θ/a</u>	<u>0.5</u>	<u>0.8</u>	<u>1.3</u>	<u>1.25</u>
<u>f</u>	<u>0.45</u>	<u>4.45</u>	<u>4.45</u>	<u>4.45</u>
M	0.87	0.87	0.87	0.87

**Iteration Chart: 4**

All the basic dependent and independent parameters are known and approximating the values up to some acceptable previous design consideration to get the next step optimal limit of design parameter.

The steering system consist of steering wheel, steering column, universal joint (b/n pinion and steering shaft), tie rod, ball joint and steering rack. But the methodology of steering design is sub categorized in two methodologies as described in the loop chart below.

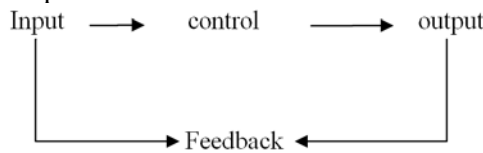
**Steering system component design**

1 .Open loop control

↓  
Switch action only  
The steering is closed feedback system in which the driver gives input and the steering system or the control system generates output in the form

2. Close loop control

↓  
ECU  
of steer angle and then the road reacts back on system and gives feedback to the driver.



**Loop Diagram: 1**

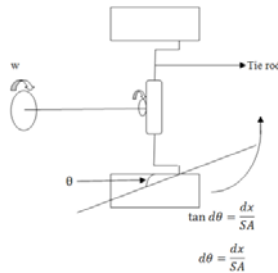
The design of steering system is done in two steps the first is steering kinematics and then the dynamics of steering is designed as per the kinematic design.

- 1 .Steering kinematics → steering ratio
- 2. Steering dynamics → steering moment

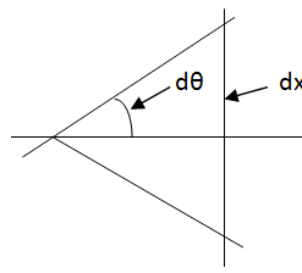
**Mathematic modeling of steering system**

Rack travel is described as the amount by which rack will travel for one complete rotation of pinion.

Let's name this parameter as "t".  
t will be a function of pinion pitch to rack pitch ratio.  
Wheel travel is defined as angle by which wheel will get steered.



**Figure 1: Steering diagram**



**Figure 2: Zoomed view of tyre displacement**

Now,  
If rack travels t, steering wheel rotates by 360 degrees.

Thus, if rack travels x steering wheel rotates by  $\frac{360x}{t}$  degrees.

So, if tire rotate by  $\frac{x}{SA}$  rad. or  $\frac{180x}{\pi \cdot SA}$  deg. then steering wheel rotates by  $\frac{360x}{t}$  deg.

So, this is our steering ratio.

Now, let's calculate values: As the older calculations have some acceptable values,  
Wheel base = 1562 mm, and the minimum radius we will cover = 7 m (7000 mm)

So,  $\frac{1562}{7000} = 12.7 \approx 13 \text{ deg (approx.)}$

We will assume steering ratio = 18:1,  
We have maximum rack travel = 40 mm,  
So, 360 → 40 mm rack travel

$13 \times 18 = 234$  deg (steering wheel angle)  
 $6.28 \times 37 = 234$  deg  
 SA =  $114.6 \approx 115$  mm,

If a man applies 100 N of force on the steering wheels,  
 And if radius of steering wheel is say  $R = 10$  cm then torque  $(T) = 10$  N-m.  
 If radius of pinion is say 1 cm then force on tie rod = 1000N.  
 If arm is 32 mm then steering torque  $(T') = 1000 \text{ N} \times 115 \text{ mm} = 315$  mm.  
 Now, if we want 20% of it as align torque then caster trail = 24.190 mm

So, the caster angle  $\tan\theta = \text{caster trail}/\text{radius of wheel} = 24.190/256$   
 $\theta = 5.455$  deg.

Now, Fluctuation in longitudinal force is = 600 N

Then, required torque is of 6.5 N-m to feel on steering,

Then scrub radius  $R_s = 10.7$  mm.

In front view joint axis provide kingpin angle & in the side view joint axis provide caster angle

For calculation of kingpin angle:

$$\tan\theta = \frac{\text{half of contact patch} - \text{scrub radius}}{\text{radius of wheel}}$$

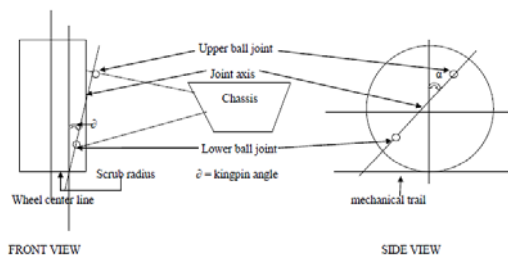


Figure 3: Front view and side view

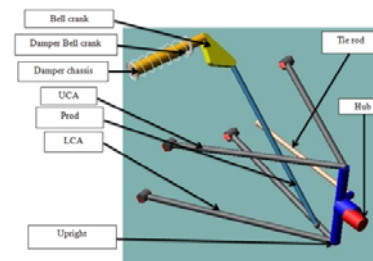


Figure 4: Descriptive view of front wishbone assembly

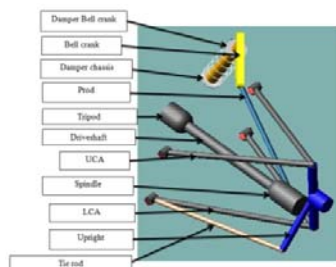


Figure 5: Descriptive view of rear wishbone assembly

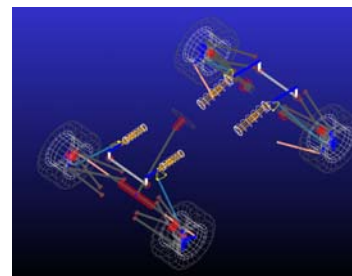


Figure 6: Rendered wireframe ISO view

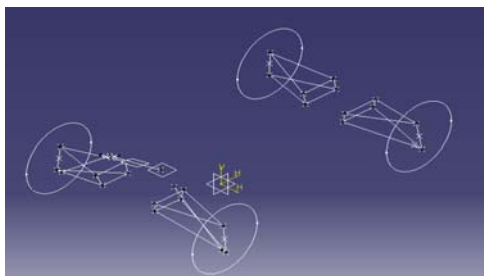


Figure 7: Wireframe ISO view

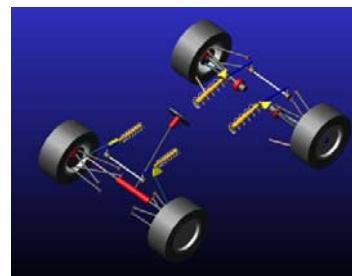
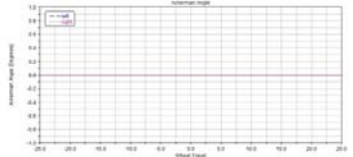
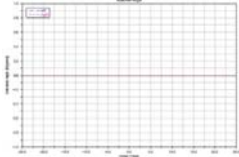
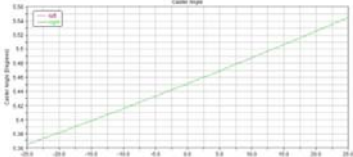

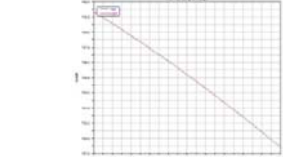
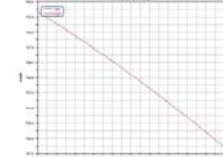
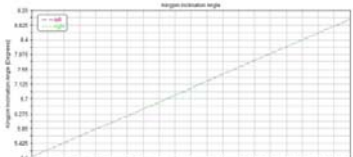
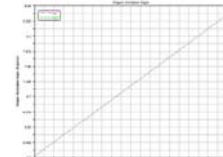
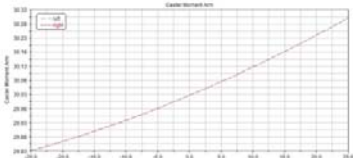
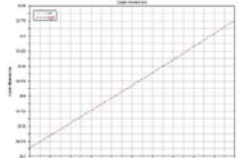
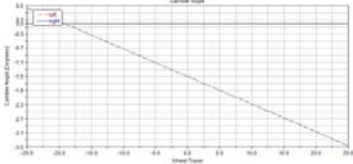
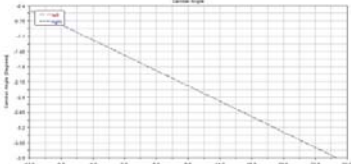
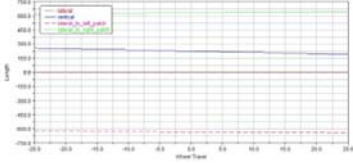
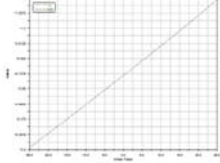
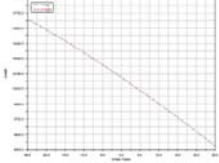


Figure 8: Rendered ISO view

Parallel wheel travel	Opposite wheel travel	Remarks
 <p><b>Figure 9: Ackerman Angle v/s Wheel Travel</b></p>	 <p><b>Figure 10: Ackerman Angle v/s Wheel Travel</b></p>	<p>Ackerann angle stability is maintained for both type of wheel travel.</p>
 <p><b>Figure 11: Caster Angle v/s Wheel Travel</b></p>	 <p><b>Figure 12: Caster Angle v/s Wheel Travel</b></p>	<p>The maximum caster angle is 5.48° for parallel wheel travel and 5.714° for opposite wheel travel both values are in considerable limit.</p>
 <p><b>Figure 13: Front View Swing Arm Length v/s Wheel Travel</b></p>	 <p><b>Figure 14: Front View Swing Arm Length v/s Wheel Travel</b></p>	<p>Change in swing arm length is directly proportional to the change in camber, kingpin and position of roll center; hence the considerable amount of change is seen for both parallel and opposite wheel travel of 25 mm bump and rebound.</p>
 <p><b>Figure 15: Kingpin Inclination Angle v/s Wheel Travel</b></p>	 <p><b>Figure 16: Kingpin Inclination Angle v/s Wheel Travel</b></p>	<p>The change of kingpin angle is measured and a linear curve is formed which shows the stability of the kingpin angle having 5.0625° at bump and 8.920° at rebound.</p>
 <p><b>Figure 17: Caster Moment Arm v/s Wheel Travel</b></p>	 <p><b>Figure 18: Caster Moment Arm v/s Wheel Travel</b></p>	<p>The overall caster moment at arm is 30.30 N-m, at parallel wheel travel and 30.775 N-m for opposite wheel travel, 25-30% of steering torque was balanced by the aligning torque to help driver by giving self returning moment for steering, so by the caster line inclination of 24.2 mm and wheel offset of 5.8 mm generates 26% self returning moment of overall caster moment.</p>

 <p><b>Figure 19: Camber Angle v/s Wheel Travel</b></p>	 <p><b>Figure 20: Camber Angle v/s Wheel Travel</b></p>	<p>The reference value of camber angle is <math>-1.5^\circ</math> and the change in camber angle is linear ranging from <math>0.487^\circ</math> of positive and <math>-3.498^\circ</math> of negative camber at parallel wheel travel and for opposite wheel travel value ranges from <math>-0.4875^\circ</math> to <math>-3.9^\circ</math>.</p>
 <p><b>Figure 21: Roll Center Location v/s Wheel Travel</b></p>	 <p><b>Figure 22: Scrub Radius v/s Wheel Travel</b></p>	 <p><b>Figure 23: Side View Swing Arm Length v/s Wheel Travel</b></p>

**OBSERVATIONS**

1. The ride frequency of vehicle is between 4 Hz to 6 Hz which is said to be ideal frequency for a performance vehicle.
2. As spring travel is 14.54 mm it shows ground clearance value will be less. And the work succeeds in minimizing the ground clearance, which is project objective.
3. Center of gravity height is 362 mm which is also again a result of good designing. And as a consequence the roll propensity will be low. Also it will minimize the effect of longitudinal load transfer.
4. Motion ratio value comes out to be 0.87 which means more load will be transfer through the wishbones than usual design. As a result, spring will bear less load transfer that's why the spring travel get reduce.

**RESULTS**

Overall ride frequency (f)	4.45 Hz	Distance between R.C. and C.G. (X')	232.76 mm
Ride rate constant (K)	46848 N/m	Torsion stiffness of chassis (K <sub>t</sub> )	48343 N-m/deg.
Overall vehicle damping constant (C)	2346 N-sec/m	Roll of chassis (θ)	1.3°
Motion ratio (M)	0.87	Contact patch of tire	220 mm
Stiffness of spring (K <sub>s</sub> )	61897 N/m	Steering angle	13°
Damper damping coefficient (C <sub>D</sub> )	2696 N-sec/m	Steering ratio (S <sub>R</sub> )	5:1
Spring travel (X)	14.54mm	Rack travel	40 mm
Height of C.G. (H)	362mm	Steering arm (SA)	32 mm
Wheel base (W)	1562.02mm	Caster angle (α)	1.2°
Track width (T)	1250mm	Kingpin angle (θ)	16.98°

**CONCLUSION**

The designing process for static conditions is completed. The numerically solved values are near approximate the simulated values hence our design procedure is correct for such kind of vehicle design. This paper includes static and dynamic parameters according to the objectives. The work successfully achieved the objective. Result implies that car designing using ADAMS has very good scope of improving vehicle geometry, behavior and performance. The overall analysis satisfies the constraints and of Formula SAE International rulebook, so the vehicle modeling under the dynamic analysis is considerable.

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