THERMODYNAMIC ANALYSIS OF NEWTONIAN AND LAMINAR FLOW ALONG AN INCLINED HEATED PLATE WITH EFFECTS OF HYDROMAGNETIC IN POROUS AND NON POROUS REGIONS

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ABSTRACT

The purpose of this work is to thermodynamic analysis of Newtonian and laminar flow along an inclined heated plate with effects of hydromagnetic and porous media in two regions. The upper surface of the liquid film is considered free and adiabatic. The effect of heat generation by viscous dissipation is included in the analysis. The influence of the viscous dissipation on velocity, temperature and entropy generation is examined.

Index Terms: Inclined plate, Thermodynamic analysis, viscous dissipation

I. INTRODUCTION

The study of flow through porous medium is important, because of its interesting application in diverse field of science, engineering and technology. The practical application are in the percolation of water through soil, extraction and filtration of oil from wells, the drainage of water, irrigation and sanitary engineering and also in the inter disciplinary field, such as Biomedical engineering etc. The lung alveolar is an example that finds application in an animal body. The classical Darcy's law Musakat (1937) states that the pressure gradient pushes the fluid against the body forces exer1ted by the medium which can be expressed as,

$$\vec{V} = -\left(\frac{K}{\mu}\right)\nabla P \quad (1)$$

The law gives good results in the situations when the flow is unidirectional or the flow is at low speed. In general the specific discharge in the medium need not be always low. As the specific discharge increase, the convective force developed and the internal stress generates in the fluid due to viscous nature as fiber glass, papers of dandelion the flow occurs even in the absence of the pressure gradient. Modification for the classical Darcy's law was considered by the Beverse & Jaseph (1967), Saffman (1971) and others. A generalized Darcy's law proposed by Brinkman (1947) is given by

$$\mu\nabla^2\vec{V} - \left(\frac{K}{\mu}\right)\nabla\vec{V} - \nabla P = 0 \quad (2)$$

where μ and K are coefficient of viscosity of the fluid and permeability of the porous medium.

The generalized equation of momentum for the flow through the porous medium is

$$\mu\nabla^2\vec{V} - \left(\frac{K}{\mu}\right)\nabla\vec{V} - \nabla P = \rho\left[\frac{\partial\vec{V}}{\partial t} + (\vec{V}.\nabla)\vec{V}\right] \quad (3)$$

The classical Darcy's law helps in studying flows through porous medium. In the case of highly porous medium such as papers of dandelion etc., the Darcy's law fails to explain the flow near the surface in the absence of pressure gradient. The Non-Darcian approach is employed to study the problem of flow through highly porous medium by several investigators. Charyalu & Ramacharyulu (1978), Charyulu (1997) and

Singh (2002) etc. studied the flow employing Brinkman Law (1947) for the flow through highly porous medium.

I1n the present problem, we are considering the effect of magnetic field on flow of a Newtonian fluid between parallel plates with porous lining. The flow becomes two layered flow one in the porous region and other in the clear region such type of a flows find application in the inter-disciplinary fields such as Biomedical engineering etc. The flow of the blood is one such application. The blood may be represented as a Newtonian fluid and the flow of blood is two layered Lightfoot (1974) and Shukla et al. (1980). The effect of magnetic field and the coefficient of porous medium and the effect of the thickness of the porous lining on the physical quantities of the fluid flow are discussed.

II. MATHEMATICAL FORMULATION

The Newtonian in compressible fluid flow is considered between two infinite parallel plates y=±h. The two parallel plates having lining of porous medium of thickness δ. The length of the plates lies parallel to the x-axis and y-axis is perpendicular to the length of the plates. The velocity of the fluid is given as (Up, 0, 0) in the porous region and (Uc, 0, 0) in the non-porous 1region. The equation of continuity is satisfied with the choice of the velocity. The equation of motion in the two regions is given by,

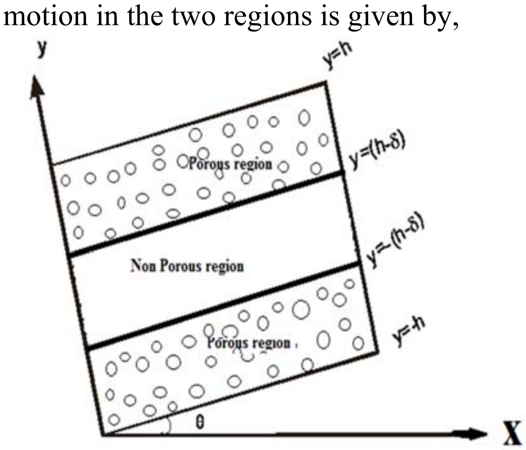

Figure : 1 Modal of the problem

The Brinkman momentum equation:

$$\mu\frac{d^2u_p}{dy^2}-\sigma B_0^2u_p-\frac{\mu}{K}u_p+\rho g\sin\theta=0;$$

$$-h\le y\le -(h-\delta) and (h-\delta)\le y\le h \quad (4)$$

$$\mu\frac{d^2u_c}{dy^2}+\rho g\sin\theta=0;$$

$$-(h-\delta)\le y\le (h-\delta) \quad (5)$$

Introducing the following dimensionless variables for the velocity and transverse distance;

$$U_p=\frac{\mu u_p}{\rho g h^2\sin\theta}, U_c=\frac{\mu u_c}{\rho g h^2\sin\theta}, Y=\frac{y}{h},$$

$$X=\frac{x}{P_e h}, P_e=\frac{\rho C_p h^3 g\sin\theta}{\mu k}, M=\frac{\sigma B_0^2}{\mu} \quad (6)$$

Where M is the magnetic parameter, P_e is the Peclet number, k is the thermal conductivity and ρ be the fluid density.

The dimensionless equation (4) becomes

$$\frac{d^2U_p}{dY^2}-r^2U_p+1=0 \quad (7)$$

Where $r^2=h^2\left(M+\frac{1}{K}\right)$

With $-1\le Y\le -(1-\frac{\delta}{h}) and (1-\frac{\delta}{h})\le Y\le 1 \quad (8)$

The dimensionless equation (5) becomes

$$\frac{d^2U_c}{dY^2}+1=0 \quad (9)$$

$$with \quad -(1-\frac{\delta}{h})\le Y\le (1-\frac{\delta}{h}) \quad (10)$$

The Boundary conditions:

$$U_c = U_p = 0 \quad at \quad Y=\pm(1-\frac{\delta}{h})$$

$$U_p = 0 \quad at \quad Y=\pm 1 \quad (11)$$

The velocity profiles are obtained by integrating equation (7) and (9) with using boundary conditions (11), we get the solutions;

$$U_p=\frac{1}{r^2}\left[1-\frac{\cosh(rY)}{\cosh(r)}\right] \quad (12)$$

$$U_c=\frac{-1}{r^2}\left[\frac{r^2}{2}\left\{Y^2-\left(1-\frac{\delta}{h}\right)^2\right\}+\frac{\cosh\left\{r\left(1-\frac{\delta}{h}\right)\right\}}{\cosh(r)}-1\right] \quad (13)$$

where U_p is the velocity for porous region and U_c is the velocity for non porous region.

III. THE STEADY-STATE THERMAL ENERGY EQUATION

$$\rho C_p u \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial y^2} \quad (14)$$

With inlet condition

$$T(0, y) = T_0 \quad (15)$$

Constant heat flux at the lower wall:

$$\frac{\partial T(x,0)}{\partial y} = -\frac{q}{k} \quad (16)$$

Adiabatic wall:

$$\frac{\partial T(x,h)}{\partial y} = 0 \quad (17)$$

Where T is the absolute temperature and T_0 is the temperature at the inlet.

Using the following dimensionless variables for temperature $\theta = \frac{k(T - T_0)}{qh}$ then the energy equation can be written in the following becomes dimensionless form

$$U \frac{\partial \theta}{\partial X} = \frac{\partial^2 \theta}{\partial Y^2} \quad (18)$$

Subject to the following conditions:

$$\theta(0,1) = 0; \frac{\partial \theta(x,0)}{\partial y} = -1; \frac{\partial \theta(x,1)}{\partial y} = 0 \quad (19)$$

To get a solution of (18), applying the method of separation of variables and we get the solutions as

Case (i): The dimensionless temperature (θ_P) for Porous region:

$$\theta_P = \lambda_1 X + \frac{\lambda_1}{r^2}\left[\frac{Y^2}{2} - \frac{\cosh(rY)}{r^2\cosh(r)} - \frac{1}{2} + \frac{1}{r^2}\right] - Y + 1 \quad (20)$$

$$\text{where} \quad \lambda_1 = \frac{r^2}{\left[1 - \frac{\tanh(r)}{r}\right]}$$

Case (ii): The dimensionless temperature (θ_c) for Non Porous region:

$$\theta_C = \lambda_2 X + \frac{\lambda_2(1-Y^2)}{24r^2}\left[\begin{array}{c}(1+Y^2) - 6\left(1 - \frac{\delta}{h}\right)^2 \\ + \frac{12}{r^2}\left\{\frac{\cosh\left\{r\left(1 - \frac{\delta}{h}\right)\right\}}{\cosh(r)} - 1\right\}\end{array}\right] - Y + 1 \quad (21)$$

$$\text{where} \quad \lambda_2 = \frac{r^2}{\left[1 - \frac{r^2}{6}\left\{1 - 3\left(1 - \frac{\delta}{h}\right)^2\right\} - \frac{\cosh\left\{r\left(1 - \frac{\delta}{h}\right)\right\}}{\cosh(r)}\right]}$$

IV. ENTROPY GENERATION RATE

According to Mahmud and Fraser [6], the entropy generation rate is define as

$$E_G = \frac{k}{T_0^2}\left[\left(\frac{\partial T}{\partial \bar{x}}\right)^2 + \left(\frac{\partial T}{\partial \bar{y}}\right)^2\right] + \frac{\mu}{T_0}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 \quad (22)$$

The dimensionless entropy generation number may be defined by the following relationship:

$$N_s = \frac{k T_0^2}{q^2} E_G \quad (23)$$

In terms of the dimensionless velocity and temperature, the entropy generation number becomes

$$N_s = \frac{1}{P_e^2}\left(\frac{\partial \theta(x,y)}{\partial x}\right)^2 + \left(\frac{\partial \theta(x,y)}{\partial y}\right)^2 + \frac{Br}{\Omega}\left(\frac{\partial u(y)}{\partial y}\right)^2$$
$$= N_x + N_y + N_f \quad (24)$$

Where the dimensionless parameters $Br = (\rho^2 g^2 \sin^2\theta h^3)/q\mu$ is the Brinkman number, $\Omega = (qh/kT_0)$ the dimensionless temperature difference. The viscous dissipation parameter ($B = Br\ \Omega^{-1}$) is defined as the product of the Brinkman number and the inverse of dimensionless temperature difference .The viscous dissipation parameter is an important dimensionless number for the irreversibility analysis. It determines the relative importance of the viscous effects for the entropy generation. N_x and N_y are the entropy generation by heat transfer due to both axial and transverse heat conduction respectively and N_f is the entropy generation due to fluid friction.

Case (i): Entropy generation rate (N_{Sp}) for Porous region:

$$Ns_p = \frac{1}{P_e^2}\left(\frac{\partial\theta_p(X,Y)}{\partial X}\right)^2 + \left(\frac{\partial\theta_p(X,Y)}{\partial Y}\right)^2 + \frac{Br}{\Omega}\left(\frac{\partial U_p(Y)}{\partial Y}\right)^2 = N_{xp} + N_{yp} + N_{fp} \quad (25)$$

Where Ns_p is the entropy generation rate per unit volume for porous region, N_{Xp} is the entropy generated due to heat transfer in the axial direction for porous region, N_{Yp} is the entropy generated due to heat transfer in the transfer direction for porous region and N_{Fp} is the entropy generated due to the fluid friction for porous region.

Case (ii): Entropy generation rate (N_{Snp}) for Non Porous region:

$$Ns_{np} = \frac{1}{P_e^2}\left(\frac{\partial\theta_{np}(X,Y)}{\partial X}\right)^2 + \left(\frac{\partial\theta_{np}(X,Y)}{\partial Y}\right)^2 + \frac{Br}{\Omega}\left(\frac{\partial U_{np}(Y)}{\partial Y}\right)^2 = N_{xnp} + N_{ynp} + N_{fnp} \quad (26)$$

Where Ns_{np} is the entropy generation rate per unit volume for non porous region, N_{Xnp} is the entropy generated due to heat transfer in the axial direction for non porous region and N_{Ynp} is the entropy generated due to heat transfer in the transfer direction for non porous region, NF np is the entropy generated due to the fluid friction for non porous region.

V. RESULT AND DISCUSSIONS

It is clear from the Figure 2 that velocity of porous region (U_P) decrease in convergent manner on increasing magnetic parameter (M).if permeability parameter is taken constant (K=1). In Figure 3 that velocity of porous region (U_P) increase in convergent manner on increasing permeability parameter (K), for constant magnetic parameter (M=1).

The velocity of non porous region (U_C) profile for different values of magnetic parameter (M) is examine in Figure 4 and for different values of permeability parameter (K) it is clear that in Figure 4 if the values of magnetic parameter (M) is increase then velocity of non porous region decrease, for constant permeability parameter (K=1) and in Figure 5, the velocity of non porous region increase on increasing permeability parameter (K).for constant magnetic parameter (M=1).

Figure 6 shows the divergent variation of temperature of porous region opposed to Y axis for different values of group parameter (r).it clear that in figure if the value of group parameter (r) is increase then the temperature of porous region is decreases with constant value of X=1. In figure 7 the temperature of porous region rises along with the increment in the different values of X for constant value of group parameter (r = 0.2). More over lines are showing variation of temperature along with X is almost parallel to each other.

In Figure 8 the temperature of non porous region rises along with the increment in the different values of X for constant value of group parameter (r = 0.2) and δ/h = 0.2. More over lines are showing variation of temperature along with X is almost parallel to each other. In Figure 9, rise of the temperature of nonporous region is noticed if value of group parameter (r) increases for constant value of X=0.2 and the height of non porous region to the height of medium bears a thickness of porous region (δ/h = 1). In Figure 10 the temperature of non porous region increases if the value of δ/h is increases for constant values of X = 0.2 and r = 0.2.

In Figure 11, the entropy generation rate of porous region (Ns_P) increases on increasing group parameter ($Br\Omega^{-1}=B$) in a very narrow region. Also initial variation of entropy generation rate of porous region (Ns_P) is almost coinciding with each other. For constant value of Pe =100 X = 0.2, r = 0.2. In Figure 12, the entropy generation rate of porous region (Ns_P) increases on increases group parameter (r) with the constant values of Pe=100 and group parameter ($Br\Omega^{-1}$= 0.2). As well as initial variations are closer and latter variations are wider.In Figure 13, the entropy generation rate of non porous region (Ns_{np}) increases on increasing the value of group parameter ($Br\Omega^{-1}=B$).

Also initial variation of entropy generation rate of non porous region (Ns_{np}) is coinciding with each other. For constant value of Pe=100, δ/h=1, r =1. The spartial distribution of the entropy generation rate of non porous region (Ns_{np}) for different values of group parameter (r) is plotted in Figure 14. It is interesting to note that entropy generation rate decreases in transverse direction and decreases with an increase in group parameter (r).it is clear in figure that the entropy generation rate of non porous region (Ns_{np}) almost same at both the terminal points of the channel. It shows that the negligible effects of magnetic field and porous permeability at terminal points of the channel. Figure 15 shows the spartial distribution of entropy generation rate of non porous region (Ns_{np}) for different values of the thickness of porous region (δ/h).for all the values of δ/h the entropy generation rate decreases in the transverse direction and increase with an increase in the values of δ/h . Figure shows the entropy generation rate of non porous region (Ns_{np}) almost same at both the terminal points of the channel. It is clear that the negligible effects of different values of δ/h at terminal points of the channel.

VI. CONCLUSION:

This paper presents the application of the second law of thermodynamics analysis of Newtonian flow along an inclined heated plate with effects of hydromagnetic and porous media in two regions. The velocity and temperature profiles are obtained and used to evaluate the entropy generation numbers for porous and non porous region. For different values of the group parameters, the entropy generation rate is function of Brinkman number as shown by the graph which gradually increases symmetrically about the centerline in the transverse direction from the lower wall towards the upper wall. This clearly implies that viscous dissipation has negligible effect on both regions i.e. Porous and Non porous region the entropy generation rate at the initial part of a channel.

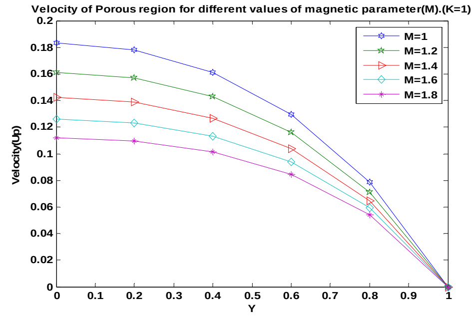

Figure 2: Velocity profile of Porous region for different values of magnetic field (M).

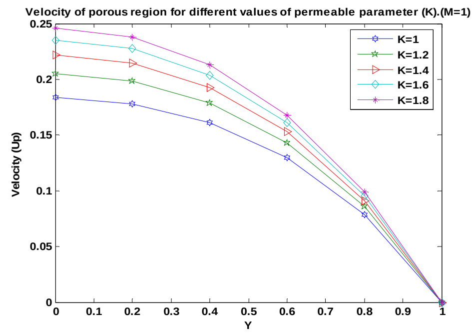

Figure 3: Velocity profile of Porous region for different values of permeable parameter (K).

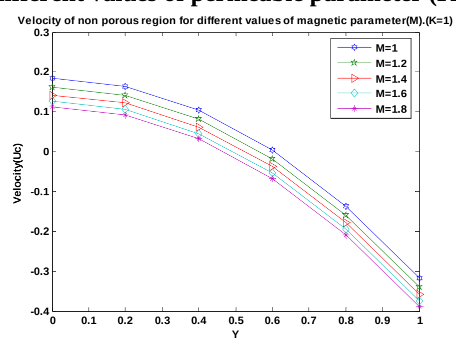

Figure 4: Velocity profile of Non Porous region for different values of magnetic field (M).

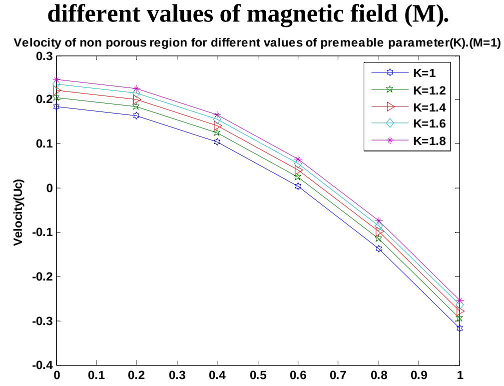

Figure 5: Velocity profile of Non Porous region for different values of permeable parameter (*K*).

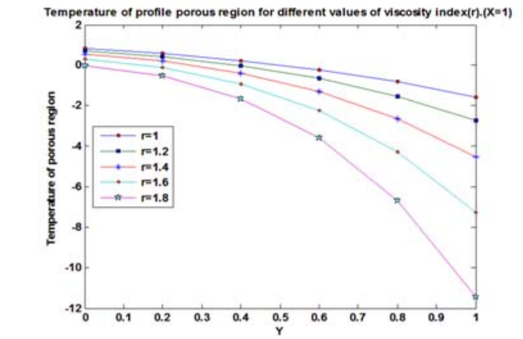

Figure 6: Temperature profile of Porous region for different values of viscosity index (r).

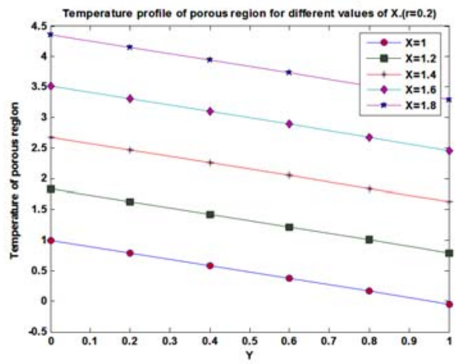

Figure: 7: Temperature profile of Porous region for different values of axial distance (X).

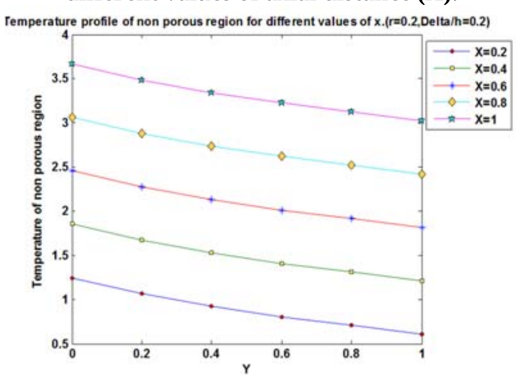

Figure: 8: Temperature profile of Non Porous region for different values of axial distance (X).

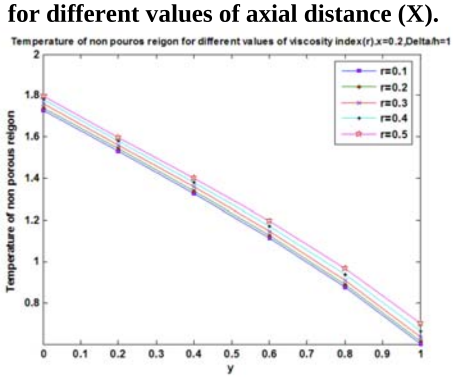

Figure: 9: Temperature profile of Non Porous region for different values of viscosity index *r*.

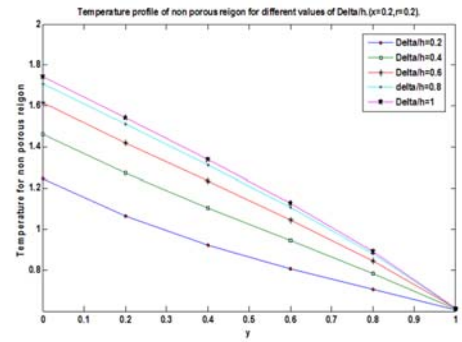

Figure: 10: Temperature profile of Non Porous region for different values of porous region thickness *δ/h.*

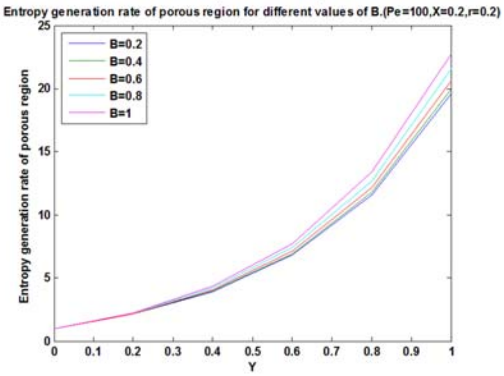

Figure: 11: Entropy generation rate profile of Porous region for different values of group parameter *B*.

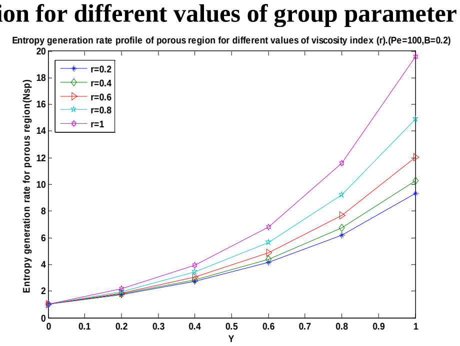

Figure: 12: Entropy generation rate profile of Porous region for different values of viscosityindex *r*.

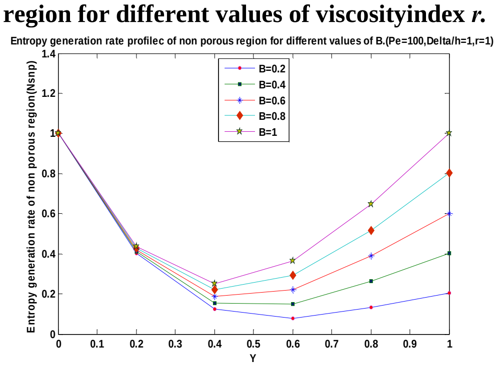

Figure: 13: Entropy generation rate profile of Non Porous region for different values of group parameter *B*.

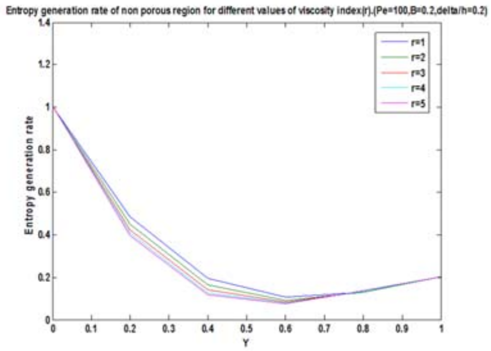

Figure: 14: Entropy generation rate profile of Non Porous region for different of viscosity index *r*.

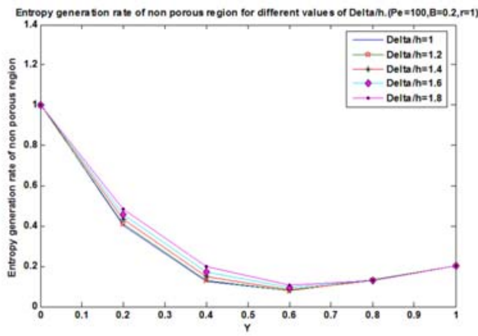

Figure: 15: Entropy generation rate profile of Non Porous region for different values of porous region thickness *δ/h*.

VII. REFERENCES

[1]. Bejan, A., Second law analysis in heat transfer. *Energy. 1980, 5, 721 732.*

[2]. Bejan, A. Entropy Generation through Heat and Fluid Flow. *John Wiley & Sons. Inc.: Canada 1994, Chapter 5, p 98.*

[3]. Bejan, A.. Entropy Generation Minimization, CRC Press Flow. *John Wiley & Sons. Inc.: Canada 1996, Chapter 5, p 98.*

[4]. Bejan, A. Entropy Generation Minimization, *C RC Press: USA* 1996.

[6]. Mahmud, S. and Fraser, R. A., The second law analysis in fundamental convective heat transfer problems. *Int. J. of Thermal Sciences 2002, 42 (2), 177-186.*

[7]. Makinde, O. D., Exothermic explosions in a s lab: A case study of series summation technique. *Int. Comm. Heat and Mass Transfer 2004, 31,8 , 1227-1231.*

[8]. Rama Subba, Reddy Gorla , David M. Pratt, Second Law Analysis of a non-Newtonian Laminar Falling Liquid Film along an Inclined Heated Plate. *Entropy, 2007, 9, 30-41.*

[9] . Sahin, A.Z. Entropy generation and pumping power in a turbulent fluid flow through a smooth pipe subjected to constant heat flux. *Exergy, an International Journal 2002, 2, 314-321.*

[10]. Woods, L.C. Thermodynamics of fluid systems, *Oxford University Press*, Oxford, 1975.