ASYMPTOTIC BEHAVIOUR OF SOLUTIONS OF MHD FLOW EQUATIONS WITH HEAT AND MASS TRANSFER DUE TO A POINT SINK

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Abstract
This Paper deals with the asymptotic behaviours of the solutions of the similarity boundary layer equations governing magneto-fluid dynamic steady incompressible laminar boundary layer flow for a point sink with an applied magnetic field, heat and mass transfer. The two-point boundary value problem governed by self-similar solutions has been solved by using the method of the asymptotic integration of second order linear differential equations. For the study of the asymptotic behaviours as the independent variable tends to infinity, some topological arguments expressed in the form of lemma and theorems have been used.
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1 Introduction

One of the most important problem in the study of differential equation and their applications is that of describing the nature of the solution for large positive values of the independent variables and this purpose is completely served by the study of asymptotic behaviours. Because of this fact, the study of the asymptotic nature of the solution of Falkner -Skan equations governing study twodimensional flow of a slightly viscous incompressible fluid past a wedge was initiated by Hartman [6] and later extended by researchers like Singh and Kumar [7], Singh and Verma [8], Singh [9] ,etc.

2 Governing Equations

Let us consider the steady laminar incompressible axisymmetric boundary layer flow of an electrically conducting fluid in a circular cane at the vertex (see Fig. 1). The hole can be regarded as a three-dimensional sink.

MHD flow problems with magnetic field find applications in vortex chambers, power generators, nuclear reactors, evolution of rotating magnetic tars, geophysical fluid-dynamic, etc. The study of the boundary layer flow of an electrically conducting fluid on a cone due to a point sink with an applied magnetic field is relevant in the study of conical nozzle or diffuser -flow problems and hence has been undertaken by Choi and Wilhelm [1]. The foregoing problem in the absence of magnetic field, mass flux diffusion and heat transfer has been studied in the part by Rosenhead [2]. The same problem was studied by Takhar [3] to include the effects of the magnetic field, mass flux diffusion and heat transfer. The problem was solved by suing method on the lines of soundalgekar et. al. [4].

The objective of the present paper is to study the asymptotic behaviours of the solutions of the equations governing the problem dealt by Takhar [3] as the independent variable tends to infinity.

The principle and the linearly independent solutions have been found based on the method of asymptotic integration of second order linear differential equations. While using the method, some required results pertaining to the existence and uniqueness of the solutions have also been discussed in the form of lemma and theorems on the lines of Gabutti [5].

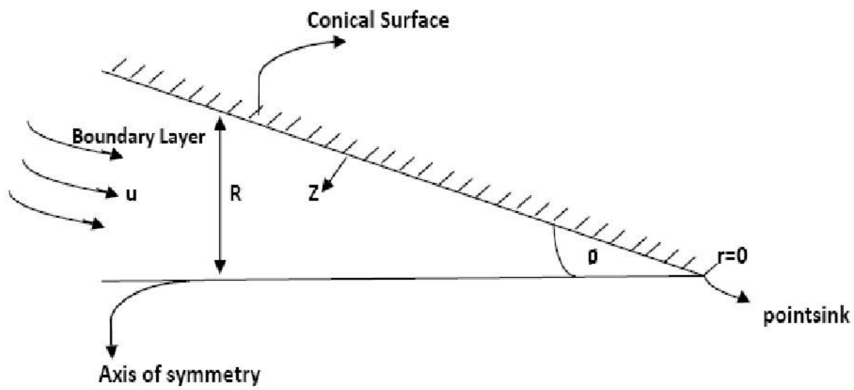

Figure 1:

A magnetic field B_0 fixed relative to the fluid is applied in Z-direction. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in compression with the applied magnetic field. The wall and the free- stream are maintained at a constant temperature and concentration. The Hall effect and the dissipation terms are neglected. The effect of mass transfer (suction and injection) has been included in the analysis. It is assumed that the injected gas possesses the same physical properties as the boundary layer gas and has a static temperature equal to the wall temperature. Both gases are assumed to be perfect gases. The boundary layer equations under the foregoing assumptions are:

$$(ru)_r + (rw)_z = 0 \tag{2.1}$$

$$uu_r + wu_z = -\rho^{-1}P_r + \vartheta u_{zz} - \rho^{-1}\sigma B_0^2 u \tag{2.2}$$

$$uT_r + wT_z = \alpha T_{zz} \tag{2.3}$$

$$uC_r + wC_z = DC_{zz} \tag{2.4}$$

where

$$-\rho^{-1}P_r = UU_r + \rho^{-1}\sigma B_0^2 U, \ \ U = -\frac{m}{r^2}, \ \ m > 0 \tag{2.5}$$

The boundary conditions are given by

$$\left\{ \begin{array}{l} u(r,0) = 0, \quad w(r,0) = w_w, \quad T(r,0) = T_w, \ C(r,0) = C_w \\ u(r,\infty) = U, \quad T(r,\infty) = T_\infty, \ C(r,\infty) = C_\infty. \end{array} \right. \tag{2.6}$$

Applying the following transformations

$$\left\{ \begin{array}{l} \eta = \frac{m^{1/2} z}{(2\vartheta r^2)^{1/2}}, \quad ru = \psi_z, \quad rw = -\psi_r, \quad \psi = -(2m\vartheta r)^{1/2} f \\ u = Uf'(\eta), \quad w = (\frac{m\vartheta}{2r^2})^{1/2}(f - 3\eta f') \\ \frac{T - T_\infty}{T_w - T_\infty} = g(\eta), \quad \frac{C - C_\infty}{C_w - C_\infty} = G(\eta) \\ M = \frac{2\sigma B_0^2 r^3}{m\rho}, \quad P_r = \frac{\vartheta}{\alpha}, \quad S_c = \frac{\vartheta}{D}, \quad f_w = w_w(\frac{2r^3}{m\vartheta})^{1/2} \end{array} \right. \tag{2.7}$$

to equations (2.1) to (2.4), we find equations (2.1) is satisfied identically and equations (2.2) to (2.4) reduce to self-similar equations given by

$$f^{000} - ff^{00} + 4(1 - f^{02}) + M(1 - f^{0}) = 0 \qquad (2.8)$$

$$g00 - Prfg0 = 0 \qquad (2.9)$$

$$G00 - ScfG0 = 0 \qquad (2.10)$$

The boundary conditions (2.6) reduce to

$$f(0) = f_w,\ f^{0}(0) = 0,\ f^{0}(\infty) = 1 \qquad (2.11)$$

$$g(0) = 1,\ g(\infty) = 0 \qquad (2.12)$$

$$G(0) = 1,\ G(\infty) = 0 \qquad (2.13)$$

Here r and z are the distance along and perpendicular to the cone; R is the radius of the cone ($R = rsin\varphi$), φ is the semi-vertical angle of the cone; u and w are velocity components along r and z direction; ψ and f are the dimensional and dimensionless stream functions; P is the static pressure; C and T are concentration and temperature; g and G are dimensionless temperature and concentration; η is similarity variables; σ, ϑ and ρ are the density, kinematic viscosity and electrical conductivity; B_0 is the magnetic field; α and D are thermal diffusivity and binary diffusion coefficient; U is the inviscid flow velocity; m is the strength of the point sink; P_r and S_c are prandtl number and Schmidt number; M is the magnetic parameter;f_w is the mass transfer parameter the subscripts r and z denote derivative w.r.t. r and z; the subscripts w and ∞ denote conditions at the wall and in the free stream; and prime denotes derivative with respect to η.

3 Mathematical Analysis

The asymptotic behaviour of solutions of (2.8), (2.11) can be studied in terms of topological arguments pertaining to the existence and uniqueness of the solutions of (2.8),(2.11) expressed in solutions, the form of following lemma and theorem.

3.1 Existence and Uniqueness

Lemma 3.1. *Let p, F be d-dimensional vectors and F(η,p) continuous on an open (η,p)- set Ω such that the solutions of initial value problems associated with*

$$p^{0} = f(\eta,p) \qquad (3.1)$$

are unique. Let Ω_0 be an open subset of Ω with the properties that all egress points from Ω_0 are strict egress points and that the set Ω_e of egress points is not connected. Let Ω_i denote the set of ingress points of Ω_0 and S a connected subset of $\Omega_0 \cup \Omega_e \cup \Omega_i$ such that $S \cap (\Omega_0 \cup \Omega_i)$ contains two points (η_1,p_1), (η_2,p_2) for which the solutions $p_j(\eta)$ of (3.1) through (η_j,p_j) for $j = 1,2$ leave Ω_0 with increasing η at points of different connected components of Ω_e. Then there exists at least one point $(\eta_0,p_0) \in S \cap(\Omega_0 \cup\Omega_i)$ such that the solution $p_0(\eta)$ of (3.1) determined by $p_0(\eta_0) = p_0$ remains in Ω_0 on its (open) right maximal interval of existence.

Proof. The proof of the Lemma 3.1 is given (Hartman [10], p.520). For the definitions of egress, ingress and strict egress point let us see Hartman ([10], p.37)

Theorem 3.1. *The boundary value problem (2.8), (2.11) has at least one solution $f = f(\eta)$ on $(0,\infty)$ such that*

$$0 < f^{0} < 1,\ \eta \in [0,\infty). \quad (3.2)$$

$$f^{00} > 0,\ \eta \in [0,\infty). \quad (3.3)$$

Proof. The proof of the theorem is based on Lemma 3.1. To this end, we rewrite the equation (2.10) in the system form. If we set $p_1 = f$, $p_2 = f^{0}$, $p_3 = f^{00}$ then we have

$$\begin{cases} p_1' = p_2 \\ p_2' = p_3 \\ p_3' = p_1p_3 - 4(1 - p_2^2) - M[1 - p_2] \end{cases} \qquad (3.4)$$

Let us define

$\Omega = \{(\eta,p_1,p_2,p_3) : \eta,p_1,p_2,p_3 \in <\}$ *and* $\Omega_0 = \{(\eta,p_1,p_2,p_3) : \eta,p_1 \in <,\ 0 < p_2 < 1,\ p_3 > 0\}$

To determine the ingress and egress points, let us introduce the following boundary sets associated with Ω_0:

$$\Omega_1 = \{(\eta,p_1,p_2,p_3) : \quad \eta,p_1 \in <,\ p_2 = 0,\ p_3 > 0\}$$
$$\Omega_2 = \{(\eta,p_1,p_2,p_3) : \quad \eta,p_1 \in <,\ 0 < p_2 < 1,\ p_3 = 0\}$$
$$\Omega_3 = \{(\eta,p_1,p_2,p_3) : \quad \eta,p_1 \in <,\quad p_2 = 1,\quad p_3 > 0\}$$
$$\Omega_4 = \{(\eta,p_1,p_2,p_3) : \quad \eta,p_1 \in <,\quad p_2 = 1,\quad p_3 = 0\}$$
$$\Omega_5 = \{(\eta,p_1,p_2,p_3) : \quad \eta,p_1 \in <,\quad p_2 = 0,\quad p_3 = 0\}$$

Here the set of ingress points is $\Omega_i = \Omega_1$. In fact in Ω_1 we have $p_2 = 0$ and $p_3 = p_2' > 0$.

The set of strict egress points is $\Omega_e = \Omega_2 \cup \Omega_3$. This follows from $p_2 = 1,\ p_3 = p_2' > 0$ for Ω_3 and from $p_2' = 0\ p_3' = -4(1-p_2^2) - M(1-p_2) < 0$ for Ω_2.

The set Ω_4 is composed of solution $p_1 = \eta + C$ (C is constant); therefore, the points in Ω_4 are neither egress nor ingress points. Thus Ω_e is not connected.

For points $(\eta,p_1,p_2,p_3) \in \Omega_5$ it holds that $p_2(\eta_0) = p_3(\eta_0) = 0$ and $p_3' = -4 - M < 0$.

These imply that $p_3(\eta),p_2(\eta) < 0$ if $|\eta-\eta_0|$ is small enough. Thus the solution (p_1,p_2,p_3) passing through $(\eta_0,p_0,0,0)$ is not in Ω_0.

Now, if k is a fixed number satisfying $0 < k < \infty$, let us set

$$S = \{(\eta,p_1,p_2,p_3) : \quad \eta = 0, p_1 = 0, p_2 = 0,\ p_3 = k\}$$

Clearly, S is connected subset of $\Omega_0 \cup \Omega_e \cup \Omega_i$.

The point $(0,0,0,k_1) \in S$, where k_1 is small and positive, is a strict ingress point of Ω_0 and the solution of (3.4) with $p_1(0) = p_2(0) = 0$, $p_3(0) = k_1$ leaves Ω_0 through the component Ω_2. Indeed, the solution of (3.4) with $p_1(0) = p_2(0) = 0$, $p_3(0) = k_1$ satisfies $p_3'(0) = -4 - M$ so that, by continuity of initial data, it follows $p_3(\eta) < 0$, $\eta > 0$ if k_1 is sufficiently small.

On the other hand, if $k_2 > 0$ is large enough, i.e. solution of (3.4) satisfying $p_1(0) = p_2(0) = 0$, $p_3(0) = k_2$ leaves Ω_0 through a point in Ω_3.

To verify this, let us note that $(\eta,p_1,p_2,p_3) \in \Omega$ implies that $p_3(\eta),p_2(\eta) > 0$ and $0 \le p_1(\eta) \le \eta$ for some $\eta > 0$. Let us use it in the third equation of (3.4) and integrate to find

$$p_3(\eta) \ge k_2 - (4 + M)\eta$$

Hence if k_2 is sufficiently large and the solution of (3.4) through $(0,0,0,k_2)$ in Ω_0 on $[0,\eta_1)$ for some $\eta_1 > 0$, then $p_3(\eta)$ is greater than a given positive constant on $[0,\eta_1)$ and such a solution leaves Ω_0 through Ω_3.

From the Lemma 3.1, it follows then that there exists a point $(0,0,0,\hat{k})$ in $S \cap (\Omega_0 \cup \Omega_1)$ such that the solution $(\hat{p}_1,\hat{p}_2,\hat{p}_3)$ of (3.2) with $\hat{p}_1(0) = \hat{p}_2(0) = 0$, $\hat{p}_3(0) = \hat{k}$, remains in Ω_0 on its right maximal interval of existence. Because of the structure of Ω_0, this is necessarily $[0,\infty)$. Finally, we prove that

$$\lim_{\eta\to\infty} \hat{p}_3(\eta) = 0 \quad \lim_{\eta\to\infty} \hat{p}_2(\eta) = 1. \quad (3.5)$$

The first limit follows immediately by observing that if we suppose, for the purpose of obtaining a contradiction, that $\lim_{\eta\to\infty} \hat{p}_3(\eta) = C \neq 0$, then we obtain $|\hat{p}_2(\eta)| > 1$, which contradicts $(\eta,\hat{p}_1,\hat{p}_2,\hat{p}_3) \in \Omega_0$ for $\eta \in (0,\infty)$.

Analogously, let us suppose if possible, that $\lim_{\eta\to\infty} \hat{p}_2(\eta) = \hat{p}_2(\infty)$ where $0 < \hat{p}_2(\infty) < 1$. The initial condition $\hat{p}_1(\eta) = 0$ and structures of the sets Ω_2, Ω_3 imply $\hat{p}_1(\eta)$ and $\hat{p}_1(\eta)\hat{p}_2(\eta) \ge 0$, $\eta \in [0,\infty)$. The use of this and the first limit of (3.5) into the third equation of (3.4) gives

$$\lim_{\eta\to\infty} \hat{p}_3(\eta) = \lim_{\eta\to\infty} [\hat{p}_1\hat{p}_3 - 4(1-\hat{p}_2^2) - M(1-\hat{p}_2)]$$
$$\ge -4\{(1-\hat{p}_1^2(\infty))\} - M)\{1-\hat{p}_2^2(\infty)\} \neq 0$$

Two integrations lead to $\lim_{\eta\to\infty} \hat{p}_2(\eta) = \infty$. This contradiction completes the proof of (3.5) are true showing that the inequalities in (3.2), (3.3) are true. Thus the proof of Theorem 3.1 is completed. □

Theorem 3.2. *There is a unique solution of the boundary value problem (2.8), (2.11) such that* $f^0 > 0$. *on* $(0,\infty)$ *is satisfied.*

Proof. Let us suppose, for the purpose of obtaining a contradiction, that there are two solutions $f_1(\eta)$ and $f_2(\eta)$ of (2.8), (2.11) such that $f_1(\eta) > 0$, $f_2(\eta) > 0$ on $(0,\infty)$. If $f_1 \neq f_2$, we assume without loss of generality that $f_1'(\eta) > f_2'(\eta)$ on $(0,\eta_0)$ and $f_1'(\eta_0) = f_2'(\eta_0)$ where $0 < \eta_0 < \infty$. Then (2.11) implies that $f_1(\eta) > f_2(\eta)$ on $[0,\eta_0)$.

If we now define $r(\eta)$ by $r(\eta) = f_1(\eta) - f_2(\eta)$, we see that

$$r(\eta), r^0(\eta) > 0 \quad (0,\eta_0), \quad r(0) = r(\eta_0) = 0 \qquad (3.6)$$

Thus, $r^0(\eta)$ has relative maximum occurring at some point $\eta_M \in (0,\eta_0)$ so that

$$r^0(\eta_M) > 0,\ r^{00}(\eta_M) = 0,\ r^{000}(\eta_M) \le 0 \quad (3.7)$$

Moreover, since either f_1 or f_2 is the solutions of (2.8), (2.11) established by Theorem 3.1, hence from $f^{00}(\eta) > 0$ on $(0,\infty)$ and the second equality of (3.7) it follows that

$$f_1''(\eta_M) = f_2''(\eta_M) > 0, \quad \eta_M \in (0,\eta_0) \qquad (3.8)$$

Therefore, from (2.8), we obtain

$$r^{000}(\eta_M) = \{f_1(\eta_M)r^{00}(\eta_M) + [4f_1(\eta_M)\} + 4f_2^{\,0}(\eta_M) + M]r^0(\eta_M) + f_2^{00}(\eta_M)r(\eta_M) \quad (3.9)$$

By using (3.5), (3.6),(3.7) and the fact that $f_1'(\eta_M), f_2'(\eta_M) > 0$ into (3.9), we find that RHS is positive whereas LHS is non-positive. This contradiction proves the non-existence of η_M and implies that $f_1(\eta) = f_2(\eta)$ on $(0,\infty)$. Furthermore, the function $r^0(\eta)$ which is positive on $(0,\eta_0)$ cannot attain maximum on $0,\infty$ but $r'(\infty) = f_1'(\infty) - f_2'(\infty) = 0$. This completes the proof Theorem 3.2. □

3.2 Asymptotic Behaviour

In this section, we shall study the asymptotic behaviour of the solution of (2.8), (2.13) will be calculated based on the asymptotic integrations of second order linear differential equations.

If $f = f(\eta)$ is the solution of (2.8), let us put

$$h(\eta) = 1 - f^0(\eta) \qquad (3.10)$$

Then $h(\eta)$ satisfies the differential equation

$$h^{00} + fh^0 - [M + 4 + 4f^0]h = 0 \qquad (3.11)$$

Differentiating (3.11) gives

$$h^{000} + fh^{00} - [M + 7f^0]h = 0 \qquad (3.12)$$

In order to eliminate the middle term in (3.11), let us put

$$h = x\exp\{-\frac{1}{2}\int_0^\eta f\,d\eta\} \quad (3.13)$$

to obtain

$$x^{00} - q(\eta)x = 0 \ (3.14)$$

where

$$q(\eta) = (4+M) + \frac{9}{2}f' + \frac{1}{4}f^2$$
$$= \frac{1}{4}f^2\left[1 + \frac{18f'}{f^2} + \frac{4(4+M)}{f^2}\right] \quad (3.15)$$

From (3.13) ,

$$q'(\eta) = \frac{1}{2}ff' + \frac{9}{2}f''$$
$$q''(\eta) = -(18 + \frac{9M}{2}) + \frac{9Mf'}{2} + \frac{37}{2}f'^2 + 5ff''$$

Since $0 < f^0 < 1$, $f^{00} > 0$ and $f^0 \sim 1$, $f \sim \eta$ as $\eta \to \infty$, there is a constants C^0 such that for large η

$$\frac{q'^2}{q^{5/2}} \le C'[\frac{f''^2}{\eta^5} + \frac{1}{\eta^3}], \quad \frac{|q''|}{q^{3/2}} \le C'[\frac{f''}{\eta^2} + \frac{1}{\eta^3}]$$

In addition, $^{R\,\infty} f^{00}d\eta$ is absolutely convergent (since $f^0 \to 1$ as $\eta \to \infty$) so that

$$\int^\infty \frac{q'^2}{q^{5/2}}d\eta < \infty \ \ and \ \ \int^\infty \frac{|q''|}{q^{3/2}}d\eta < \infty \qquad (3.16)$$

$$\int^\infty \frac{f''^2}{\eta^5}d\eta < \infty$$

provided that

(3.17)

It is easy to check the validity of (3.15), for an integration by parts (integrating f^{00} and differentiating f^{00}/η^5) gives

$$\int \frac{f''^2}{\eta^5}d\eta = \frac{f'f''}{\eta^5} + \int \frac{f'}{\eta^5}[-ff'' + 4(1-f^2) + M(1-f') + \frac{5f''}{\eta}]d\eta$$

By (2.8). The last integral is absolutely convergent and liminf $f^{00}(\eta) = 0$ as $t\eta \to \infty$. Thus (3.17) holds.

Consequently, (3.16) holds, and thus (3.14) has a principal solution $x(\eta)$ satisfying, as $\eta \to \infty$,

$$x \sim c_1 q^{-1/2}(\eta)\exp(-\int^\eta q^{1/2}(s)ds), \qquad (3.18)$$

where c_1 6= 0 is a constant, while linearly independent solutions satisfy

Z η (3.19)

$x \sim c_1 q^{-1/2}(\eta)\exp(\quad q^{1/2}(s)ds);$

(cf. Exercise XI 9.6 Hartman [10], p.382).

From the last part of (3.13) and $f \sim \eta$,

$$q^{1/2}(\eta) = \frac{1}{2}f + \frac{M+4}{4} + \frac{9}{2}(\frac{f'}{f}) + O(\frac{1}{\eta^3}); \quad q^{1/4}(\eta) \sim (\frac{\eta}{2})^{1/2},$$

Hence

$$\int^{\eta} q^{1/2}(\eta)d\eta = \frac{1}{2}\int^{\eta} f d\eta + \frac{9}{2}\log f + (M+4)\int^{\eta}(\frac{d\eta}{f}) + c^0 + o(1),$$

where c^0 is a constant. Thus (3.18), (3.19) become

$$x \sim c_1\eta^{-5}\exp\left[-\int^{\eta}\left(\frac{1}{2}f + \frac{M+4}{f}\right)d\eta\right],$$

$$x \sim c_1\eta^{4}\exp\left[\int^{\eta}\left(\frac{1}{2}f + \frac{M+4}{f}\right)d\eta\right].$$

In view of (3.13), the equation (3.11) has a principal solution satisfying

$$L_1 \equiv h \sim c_1\eta^{-5}\exp\left[-\int^{\eta}\left(f + \frac{M+4}{f}\right)d\eta\right], \quad , c_1 \neq 0, \quad (3.20)$$

while the linearly independent solutions satisfy

$$L_2 \equiv h \sim c_1\eta^{4}\exp\left[\int^{\eta}\left(\frac{M+4}{f}\right)d\eta\right], \quad c_1 \neq 0, \quad (3.21)$$

as $\eta \to \infty$.

By treating (3.12) as a second order equation for h^0 in the same way that (3.11) was handled, it is seen that (3.12) has the principal solutions satisfying,

$$L_1' \equiv h' = c_1'\eta^{-8}\exp\left[-\int^{\eta}\left(f + \frac{M}{f}\right)d\eta\right], \quad c_1' \neq 0, \quad (3.22)$$

and the linearly independent solutions satisfy

$$L_2' \equiv h' = c_1'\eta^{7}\exp\left[\int^{\eta}\left(\frac{M}{f}\right)d\eta\right], \quad c_1' \neq 0, \quad (3.23)$$

as $\eta \to \infty$.

If (3.13) satisfies (3.20), then since $f \sim \eta$, it follows that R $^\infty$ $h\eta d\eta < \infty$; thus

$$f = \eta + c_2 + o(1), \qquad \int^{\eta} f d\eta = \frac{\eta^2}{2} - c_2\eta + c_3 + o(1)$$

as $\eta \to \infty$.

Substituting this into (3.20), (3.22) gives

$$1 - f' \sim c_0\eta^{-9-M}\exp\left(-\frac{\eta^2}{2} + c_2\eta\right), \quad f'' \sim \eta(1-f'), \quad (3.24)$$

as $\eta \to \infty$ where $c_0 > 0$, c_2 are the constants.

Similarly, from (3.21), (3.23) following result

$$1 - f^0 \sim c_0\eta^{8+M}, f^{00} \sim \eta^{-1}(1-f^0), \quad (3.25)$$

$\eta \to \infty$

Similarly (2.9) and (2.12) has the principal solutions

$$M_1 \equiv g \sim c_2', \quad c_2' \neq 0, \quad (3.26)$$

and that the linearly independent solutions satisfy

$$M_2 \equiv g \sim c_2'\eta^{-1}, \quad c_2' \neq 0, \quad (3.27)$$

as $\eta \to \infty$. Finally (2.10) , (2.13) has the principal solutions

$$N_1 \equiv G \sim c_3', \quad c_3' \neq 0, \quad (3.28)$$

while the linearly independent solutions satisfy

$$N_2 \equiv G \sim c_3'\eta^{-1}\exp\left(S_c\int^{\eta} f d\eta\right), \quad c_3' \neq 0, \quad (3.29)$$

as $\eta \to \infty$.

4 Results

The study of the asymptotic behaviour pays particular attention towards a desired problem for finding conditions under which a solution approaches zero as the independent variable tends to infinity, or is very small for all independent variables, or is bounded as the independent variable tends to infinity. The asymptotic behaviour of (3.24) and (3.25) can be understood from the following Theorem:

Theorem 4.1. *Let $f(\eta)$ be solution of (2.8),(2.11). Then there exists constants $c_0 > 0$, c_2 such that (3.24), holds as $\eta \to\infty$.*

Proof. For a given $f(\eta)$, it has to be decided whether $h = 1 - f^0(\eta)$ satisfies (3.20), (3.22) or (3.21) , (3.23). But (3.21) can not hold as $M > 0$, for otherwise $f = 1-f^0 \to 0$ as $\eta \to\infty$ fails to hold. Thus (3.20), (3.22) are valid and, as was seen, this gives (3.24). So the relations given in(3.24) holds as $\eta \to\infty$.

But the relations given in (3.25) do not hold as $\eta \to\infty$. Likewise, it can be seen that the principal solution (3.26) holds as $\eta \to\infty$,because the constant $C_2' \neq 0$ has a finite value. On the contrary, the linearly independent solutions given by (3.27) do not holds as $\eta \to\infty$ whereas (3.29) do not.

As a matter of above facts, it can be concluded that the principal solutions given by (3.24) , (3.26) and (3.28) exhibit asymptotic behaviour, whereas the linearly independent solution given by (3.25),(3.27) and (3.29) do not, as $\eta \to\infty$. □

5 Concluding Remarks

The asymptotic integration method to find out the solutions of non-linear boundary layer equations is the corner-stone of fluid Mechanics. This is the method to find the approximate solutions of nonlinear, non-homogenous boundary layer equations with very high accuracy for very large values of the independent variables. Thus, the main objective of the method here is to provide reasonably accurate expression for the solution for large values of independent variable. By doing this, we have succeeded in understanding the physics of the problem also. These solutions can be used to obtain the more efficient numerical procedures for computing the solution.

From the expressions it is obvious that the principal solutions are convergent in nature, whereas the linearly independent (non-principal) solutions are divergent. So the principal solutions will be o great significance for deciding the nature of the velocity and temperature profiles even for tremendous large values of the independent variable. Thus the principal solutions will provide much insight to computational methods in understanding the physics of the problem and accordingly in deciding the correctness of the solutions obtained by appropriate software or computer. Also the principal and non-principal solutions will satisfy

$$\text{(i)}\quad \frac{L_1(\eta)}{L_2(\eta)} \to 0;\ \frac{L_1'(\eta)}{L_2'(\eta)} \to 0;\ \frac{M_1(\eta)}{M_2(\eta)} \to 0 \text{ and } \frac{N_1(\eta)}{N_2(\eta)} \to 0 \text{ as } \eta \to \infty,$$

$$\text{(ii)}\quad \int^\infty \frac{d\eta}{L_1^2(\eta)} = \infty;\ \int^\infty \frac{d\eta}{L_2^2(\eta)} < \infty;\ \int^\infty \frac{d\eta}{M_1^2(\eta)} = \infty;\ \int^\infty \frac{d\eta}{M_2^2(\eta)} < \infty;\ \int^\infty \frac{d\eta}{N_2^1(\eta)} = \infty;\ \int^\infty \frac{d\eta}{N_2^2(\eta)} < \infty$$

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