NUMERICAL METHODS FOR ENGINEERING PROBLEMS: A BRIEF REVIEW

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Abstract
Compliant members inflexible link mechanisms undergo large deflections when subjected to external loads. Because of this fact, traditional methods of deflection analysis do not apply. Since the nonlinearities introduced by these large deflections make the system comprising such members difficult to solve, parametric deflection approximations are deemed helpful in the analysis and synthesis of compliant mechanisms. This is accomplished by representing the compliant mechanism as a pseudo-rigid-body model. A wealth of analysis and synthesis techniques available for rigid-body mechanisms thus become amenable to the design of compliant mechanisms. In this paper, a pseudo-rigid-body model is developed and solved for the tip deflection of flexible beams for combined end loads. A numerical integration technique using quadrature formulae has been employed to solve the large deflection Bernoulli-Euler beam equation for the tip deflection. Implementation of this scheme is simpler than the elliptic integral formulation and provides very accurate results.
Key words: Differential equations, boundary conditions, numerical tool

Introduction:

A pseudo-rigid-body model for a flexible beam can be developed based on the deflection curve of the beam tip subjected to given loads. Deflection curves for flexible beams can be obtained by solving the exact form of the Bernoulli-Euler beam equation which states that the bending moment at any point on the beam is proportional to its curvature.
Numerous techniques are available that take into account the nonlinearities introduced in the beam equation due to large deflections. A classical solution involves the solution of a second order nonlinear differential equation using elliptic integrals of the first and the second kinds (Bisshopp and Drucker, 1945; Frisch-Fay, 1963; Mattiasson, 1981; Howell and Midha, 1995). Though the technique yields a closed form solution which is exact, the involved derivations are cumbersome and time consuming. Moreover, the use of this technique is limited to relatively simple geometries and loading. Numerical analysis methods, such as finite element analysis on the other hand, are capable of solving more general problems although they provide approximate solutions.

Many other applications of compliant mechanisms may involve flexible beams subjected to both end forces and end moments, or combined loads in general. Developing a more general formulation for flexible beams with combined loads is the purpose of this work. A numerical integration technique is employed to solve deflection equations. The technique proves to be simpler in implementation than the elliptic integral formulation and provides nearly accurate results.

Deflection Equations for Flexible members with Combined End Loads

Using the Euler-Bemoulli equation for vertical force, and

The Euler–Bernoulli equation describes the relationship between the beam's deflection and the applied load:[5]

One-dimensional Golden Section Method (Rao, 1984) has been employed as an optimization scheme to determine an optimal value of y that maximizes the pseudo-rigid-body angle, ©, for individual deflection curves (i.e., for each combination of n and /c). In each case, the maximum value of the error in approximation is less than 0.5 percent. Figure 4 shows the variation of y with K for different values of n. Initially, for each n, there is a sharp decrease in 7 as K increases. This means that the pivot is displaced towards the tip in its undeflected position and that there is a decrease in the characteristic radius of the curve. For larger values of K:, y converges to a constant value of about 0.74. It is also observed that for a constant /c, variation of y with respect to n is not as significant when compared with that of K for constant n. This means that the variation of the characteristic radius factor is predominantly dependent on load ratio, K Because the beam tip initially follows a circular path for combined loads and that both the location of the pivot and the characteristic radius change with the load ratio, parameter K can be used to develop a physical model shown in Fig. 5 to approximate beam tip deflection. The model comprises three links, namely, a fixed link, a collar that slides along the fixed link and an axially compressible link that rotates about the pivot on the collar. The movement of the collar accounts for the displacement of the pivot. Since this movement is dependent on K, a pseudo force of magnitude K is introduced on the collar. A spring of undeflected length (1 - y^)/ (7„ is the value of the characteristic radius factor for «: = 0) and stiffness coefficient, Ki, is introduced to restrain the displacement of the pivot when subjected to a pseudo force, K. Another pseudo force of magnitude K is introduced at the tip of the axially compressible link to account for the decrease in the characteristic radius. Since this decrease is equal to the displacement of the pivot, the rotating link will have the same stiffness characteristics as the spring attached to the collar. Here, the uncompressed length of the rotating link is equal XayJ,. K torsional spring of stiffness, Kg is placed at the pivot to represent resistance of the rotating link against combined loads. pseudo-rigid-body representation in a complex plane is shown in Fig. lO(fo). The prescribed path or the Precision points, P|, P2, P-i and P4 and the corresponding actuation loads, Fx F2, F3 and F4 are known a priori. Here, the force applied is a follower load for which the point of application is fixed with respect to the beam tip. The unknowns in the problem are the positions, orientations and dimensions of the compliant and the rigid links. In Fig. lO(fo), Zo and a describe the initial position and orientation of the flexible segment with respect to the origin, O. Zi'is the position vector corresponding to the location of the collar or the pivot joint which can be determined using Eqs. (23) and (30). The magnitude, \Z, - Z„\ + IZ2 - Z,| is the length, L of the flexible beam where Z2 is the position vector representing the beam tip. The angle, 0 is the pseudo-rigid-body angle which can be computed using Eq. (26). Z3, Z4 and Z5 are the position vectors representing the rigid links as shown. The angles, 4' and (j) are the rigid body rotations for the coupler, Z2Z3Z5 and the output link, Z4Z5. The example is solved in two steps. The first step involves the synthesis of the dyad comprising the flexible link and the coupler. Loop closure equations for this dyad can be written as Deflections at specific points on a beam must be determined in order to analyze a statically indeterminate system , The curve that is formed by the plotting the position of the centroid of the beam along the longitudal axis is known as the elastic curve, Supports which resist a force, such as a pin, restrict displacement l Supports which resist a moment, such as a fixed end support, resist displacement and rotation or slope, We can derive an expression for the curvature of the elastic curve at any point where ρ is the radius of curvature of the elastic curve, Since we have a function for M along the beam we can use the expression relating the moment and the deflection, shear, bending moment, slope, and deflection curves identifying the maximum, minimum, and zero points for each curve .

Conclusions:

A pseudo-rigid body model is been developed to determine large beam tip deflections of flexible beams subjected to combined end loads. End vertical force, end horizontal force and a positive end moment are considered. Gauss-Chebyshev quadrature formulae is used as a numerical integration technique to solve deflection equations. Implementation of this scheme is much simpler compared to the elliptic integral formulation and provides solutions that are accurate. For load cases comprising end forces (end vertical and end horizontal forces), the characteristic radius (or the pin joint location) is been determined to be 0.81/ as compared to that of 0.85/ with the elliptic integral formulation. Here, / is the beam length. The error incurred is

4.7 percent. For combined end , loads, the value of characteristic radius is found to lie in the range of 0.73/ and 0.85/ . As the magnitude of the end moment increases, the value of characteristic radius converges to 0.74/. This is consistent with the results found by Howell . For load cases comprising end forces, the stiffness of the torsional spring, K^ was found to be 2.52 compared to 2.65 obtained using the elliptic integral formulation. This is an error of 4.9 percent. With an increase in the magnitude of the end moment, the value of the stiffness decreases and converges to a value of 1.52. The results match well with cases investigated by Howell for combined end loads using the elliptic integral formulation. An example is finally presented to illustrate the synthesis of compliant mechanisms using this model

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- Extends their assistance to HODs in the departmental activities as per the superior's instruction

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