FREE CONVECTION FLOW IN THE STAGNATION POINT REGION OF A THREE DIMENSIONAL BODY WITH VARIABLE VISCOSITY

C. Poornima[1], A.T. Eswara[2]
[1] Department of Mathematics, P.E.S. College of Engineering, Mandya, India
[2] Department of Mathematics, GSSS Institute of Engineering and Technology for Women, KRS Road, Mysore, India.
Email: poorniachar@gmail.com[1], lens15@gsss.edu.in[2]

Abstract
An analysis is performed to study the effect of variable viscosity on the steady free convection flow of a laminar incompressible viscous fluid near the stagnation point region of a three-dimensional body. The system of coupled non-linear partial differential equations governing the semi-similar flow is solved numerically using an implicit finite difference scheme along with quasilinearization technique. It is found that, the results pertaining to variable fluid properties differ significantly, from those of constant fluid properties. Further, it is observed that the temperature difference between the wall and the fluid has a pronounced effect on heat transfer.
Keywords: Heat transfer, Natural Convection, Skin friction, Three dimensional stagnation point, Variable viscosity.

1. INTRODUCTION

The thermo physical properties of fluid often vary significantly with temperature under the circumstances where large or moderate temperature gradient exists across the fluid medium. Due to the increase of temperature, molecular interchange, attractive and cohesive forces between the molecules contribute to the viscous shear stress in liquids, causing reduction in the viscosity across the momentum boundary layer, leading to a local increase in the transport phenomena. On the other hand, the effect of variation of temperature on the density of liquids (such as water) is generally considered insignificant in the fluid flow. The specific heat changes due to temperature variations are usually not important, except where large pressure changes are involved. In the flow of water at moderate temperatures, where there is no phase change, the latent heat effects can be neglected. Different studies have [1] – [3] been reported with variable viscosity and Prandtl number for two-dimensional and axisymmetric bodies.

The stagnation point flows are classic problems in the field of fluid dynamics. The problem of flow and heat transfer at a general three-dimensional stagnation point region has important applications in many manufacturing processes in petrochemical industries, the aerodynamic of plastic sheet, solar central recievers exposed to wind currents etc. The boundary layer flows near a three-dimensional stagnation point of attachment on an isothermal surface has been examined several times in the past. Poots [4] formulated the boundary-layer equations for the free convection flow at three-dimensional lower stagnation point on a general curved isothermal surface. Banks [5] concluded that the three-dimensional solution can be exhibited at a two-dimensional stagnation point for a Prandtl number $Pr = 0.72$. Sharidan et.al [6] have investigated the steady free convection boundary layer flow in the region of the stagnation point of a three-dimensional body.
The aim of the present investigation is to study the effect of variations of viscosity with temperature on the steady free convection

boundary layer flow at a three dimensional stagnation point. The fluid considered here is water ($\Pr = 7.0$) as it is one of the most common working fluids differential equations governing the semi-similar flow have been solved with the help of an implicit finite difference

2. PROBLEM ANALYSIS

Consider the steady free convection flow near the stagnation point of a three-dimensional body placed in a viscous fluid [See Fig.1]. The temperature of the surface of the body T_w is uniform and higher than the free stream temperature T_∞ $(T_w > T_\infty)$. The fluid is assumed to flow with moderate velocities, and the temperature difference between the wall and the free stream is small $(< 40^0 C)$. In the range of temperature considered (i.e., $0 - 40^0 C$), the variation of both density (ρ) and specific heat (c_p), of water, with temperature is less than 1% (see Table I) and hence they are taken as constants. However, since the thermal conductivity (k) and viscosity (μ) variation with temperature is quite significant, the viscosity is assumed to vary as an inverse linear function of temperature:

$$\mu = \frac{1}{(b_1 + b_2 T)} \tag{1}$$

Where $b_1 = 53.41$ and $b_2 = 2.43$. The numerical data for the above relation is taken from Vargaftik [7].

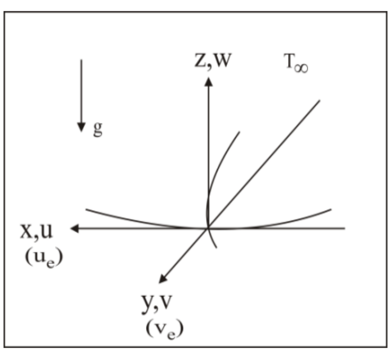


Fig. 1: Physical model and co-ordinate system

TABLE I: Values of thermophysical properties of water at different temperatures [7]

Temperature (T)(^{0}C)	*Density (ρ) (g/cm^3)*	*Specific heat (c_p) (J.10^7/kg K)*	*Thermal conductivity (k) (erg.10^5/cm. s-K)*	*Viscosity (g.10^{-2}/cm-s)*
0	**1.00228**	**4.2176**	**0.5610**	**1.7930**
10	**0.99970**	**4.1921**	**0.5800**	**1.3070**
20	**0.99821**	**4.1818**	**0.5984**	**1.0060**
30	**0.99565**	**4.1784**	**0.6154**	**0.7977**
40	**0.99222**	**4.1785**	**0.6305**	**0.6532**
50	**0.98803**	**4.1806**	**0.6435**	**0.5470**

As the fluid is incompressible, the contribution of heating due to compression is very small and it has been neglected. It is assumed that the injected fluid posses the same physical properties as the boundary layer fluid. Under these assumptions, the boundary layer equations governing steady free convection flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \nu\frac{\partial^2 u}{\partial z^2} + g\beta ax(T - T_\infty) \tag{3}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \nu\frac{\partial^2 v}{\partial z^2} + g\beta by(T - T_\infty) \tag{4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha\frac{\partial^2 T}{\partial z^2} \tag{5}$$

The boundary conditions are:

$$\begin{aligned} &u = v = w = 0, \quad T = T_w \quad for\ z = 0, x \ge 0, y \ge 0 \\ &u = v = w = 0, \quad T = T_\infty \quad for\ z \to \infty, x \ge 0, y \ge 0 \\ &u = v = w = 0, \quad T = T_\infty \quad for\ z \to \infty, x = 0, \\ &u = v = w = 0, \quad T = T_\infty \quad for\ z \to \infty, x \ge 0, y = 0 \end{aligned} \tag{6}$$

Introducing the following transformations

$$\begin{aligned} &\eta = (Gr)^{1/4} az;\ f' = F(\eta);\ \ s' = S(\eta); \\ &u = \nu a^2 x (Gr)^{1/2} F(\eta); \\ &v = \nu a^2 cy (Gr)^{1/2} S(\eta); \\ &w = -\nu a (Gr)^{1/4} (f + cs); \\ &G(\eta) = (T - T_\infty)/(T_w - T_\infty); \\ &Gr = g\beta(T_w - T_\infty)/a^3 \nu. \end{aligned} \tag{7}$$

to Eqns. (2)-(5), we see that the continuity equation (2) is identically satisfied and the Eqns (3) - (5) reduce, respectively, to

$$(NF')'+(f+cs)F'-F^2+G=0 \tag{8}$$

$$(NS')'+(f+cs)S'-cS^2+G=0 \tag{9}$$

$$G''+\Pr(f+cs)G'=0 \tag{10}$$

$$N=\mu/\mu_\infty=(b_1+b_2T_\infty)/(b_1+b_2T)=1/(1+a_1G);$$

Where

$$a_1=[b_2/(b_1+b_2T_\infty)]\Delta T_w;$$

$$\Pr=\frac{\nu}{\alpha}\ ;\ f=\int_0^\eta F\,d\eta\ ; \tag{11}$$

The transformed boundary conditions are:

$$\begin{aligned} &F=S=0,\ G=1 \quad at \quad \eta=0\\ &F=S=0,\ G=0 \quad as \quad \eta\to\infty \end{aligned} \tag{12}$$

It may be noted that, for constant fluid properties, where $N=1$, Eqns. (8) - (10) reduce to the following set of equations:

$$F''+(f+cs)F'-F^2+G=0 \tag{13}$$

$$S''+(f+cs)S'-cS^2+G=0 \tag{14}$$

$$G''+\Pr(f+cs)G'=0 \tag{15}$$

Which are exactly same as those of Banks [5].

The physical quantities of practical interest in this problem are skin friction and heat transfer coefficients, and they are defined in the form of local Nusselt number as:

$$C_{fx}=\mu\left(\frac{\partial u}{\partial z}\right)_{z=0}\Big/\left(\rho\nu^2a^3x\right)=(Gr)^{3/4}F'_w\ ; \tag{16}$$

$$C_{fy}=\mu\left(\frac{\partial v}{\partial z}\right)_{z=0}\Big/\left(\rho\nu^2a^3y\right)=(Gr)^{3/4}S'_w \tag{17}$$

$$Nu=-a^{-1}\left(\frac{\partial T}{\partial z}\right)_{z=0}\Big/(T_w-T_\infty)=-(Gr)^{1/4}G'_w \tag{18}$$

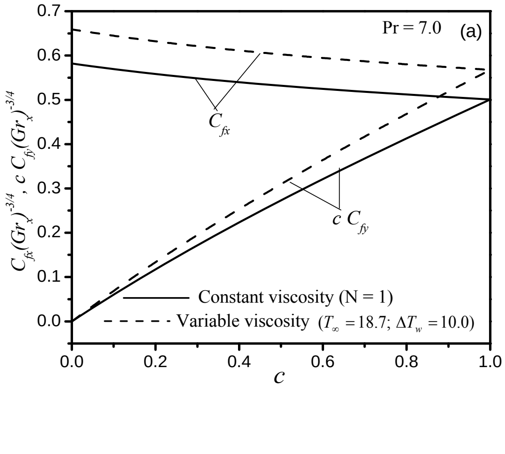
3. RESULTS AND DISCUSSION

The set of partial differential equations (8)-(10) governing the semi-similar flow subject to the boundary conditions (12) have been solved numerically using implicit finite difference scheme along with quasilinearisation technique. For the sake of brevity, the numerical method used is not presented here as it is described in detail in [8]. Numerical calculations are carried out for skin friction coefficients and velocity profile in *x*- and *y*-directions, heat transfer coefficient and temperature profile, to see effect of variable viscosity with temperature for the viscous fluid water (Pr = 7.0). Further, as most shapes of practical interest lie between cylinder $(c=0)$ and sphere $(c=1)$, we confine our computations in the range of $'c'$ where, $0\le c\le 1$. In order to assess the accuracy of the present method, the skin friction and heat transfer coefficients for constant fluid properties (i.e., $N=1$) have been compared with those of Banks [5] and Sharidan [6] and they are found to be in excellent agreement [See Table II]

The variation of skin friction coefficients and heat transfer coefficient $[C_{fx}(Gr)^{-3/4}, C_{fy}(Gr)^{-3/4}, Nu_x(Gr)^{-1/4}]$ with the constant $'c'$ are presented in Fig. 2. It is observed that as c increases $(0\le c\le 1)$, $C_{fx}(Gr)^{-3/4}$ decreases whereas $C_{fy}(Gr)^{-3/4}$ and $Nu(Gr)^{-1/4}$ increases. Further, it is found that the effect of variable viscosity is to increase both skin friction coefficients and heat transfer coefficients. In fact, the percentage of difference between constant and variable fluid properties in the case of $C_{fx}(Gr)^{-3/4}$ and $C_{fy}(Gr)^{-3/4}$ are respectively, 7.06% and 3.68% and in the case of $Nu_x(Gr)^{-1/4}$ is about 19.66% at $c=0.5$.

TABLE II: Comparision of constant fluid property ($N=1$) results with those of Banks [5] and Sharidan et. al [6]

	PRESENT RESULTS		**BANKS[5]**		**SHARIDAN ET.AL.[6]**	
c	**0**	**1**	**0**	**1**	**0**	**1**
F'_w	**0.8542**	**0.7646**	**0.8563**	**0.7611**	**0.8559**	**0.7646**
cS'_w	**0**	**0.7646**	**0**	**0.7606**	**0**	**0.7646**
$-G_w$	**0.3748**	**0.4620**	**0.3727**	**0.4611**	**0.3741**	**0.4622**



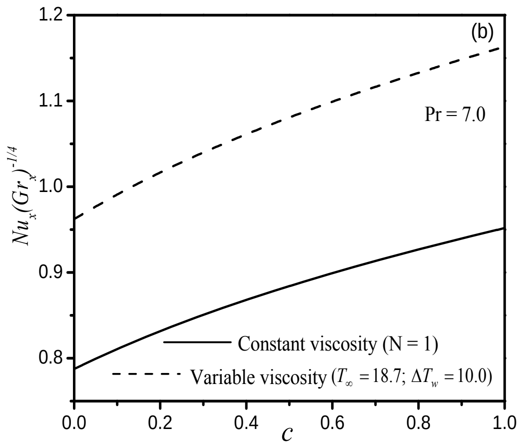
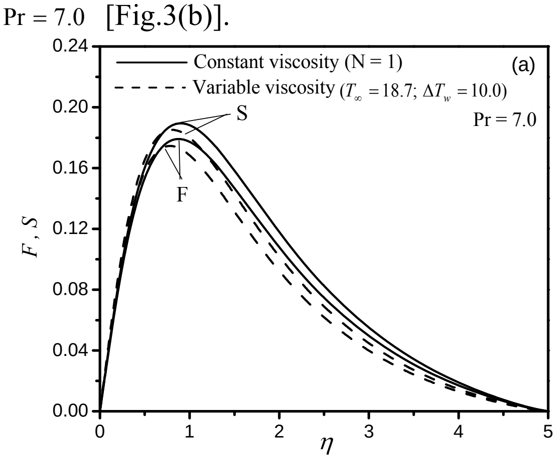


Fig. 2. Comparison of variable viscosity (μ) with constant viscosity for various values of 'c' (a) skin friction and (b) heat transfer coefficients

The velocity profiles in x- (F) and y- direction (S) and the temperature profile (G) for constant and variable viscosity when $c = 0.5$ are shown in Fig.3. It is observed that the effect of variable viscosity increases both the velocities (F) and (S) near the wall which results in the reduction of momentum boundary layer thickness. On the other hand, the effect of variable viscosity decreases the temperature profiles (G) as a result there is a reduction in the thermal boundary layer thickness. The changes in velocity profile (F) reveal the typical velocity profiles for natural convection boundary layer flow, i.e., the velocity is zero at the boundary wall then the velocity increases to the peak values and finally the velocity approaches to zero (the asymptotic value) [Fig.3(a)]. The changes in temperature profile (G) also shows the typical temperature profile for natural convection boundary layer flow as explained above when $c = 0.5$ and $\Pr = 7.0$ [Fig.3(b)].



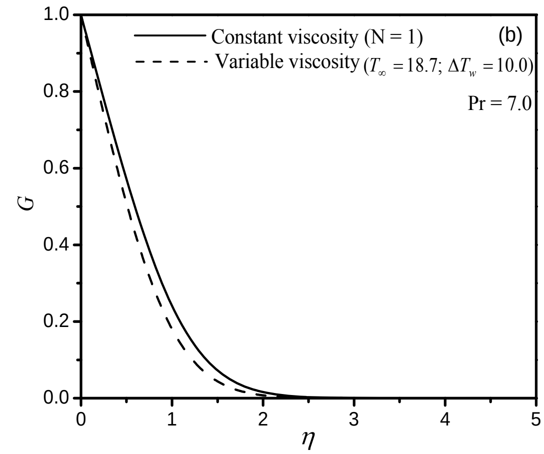


Fig. 3. (a) Velocity and (b) temperature profiles for the comparison of variable viscosity (μ) and constant viscosity.

To see the effect of the difference in the temperature (ΔT_w) between the wall and the fluid, which actually causes the variation of viscosity across the boundary layer ($\Delta T_w \approx 0$ effectively means constant viscosity), the skin friction $[C_{fx}(Gr)^{-3/4}, C_{fy}(Gr)^{-3/4}]$ and heat transfer coefficients $[Nu(Gr)^{-1/4}]$ have been plotted against [Fig. 4(a)]. Since $T_\infty = 18.7^0 C$, the maximum value of ΔT_w taken is $20^0 C$ so as to keep the temperature within the allowed value ($40^0 C$) . It is clear from the figure that both [$C_{fx}(Gr)^{-3/4}, C_{fy}(Gr)^{-3/4}$]and $Nu(Gr)^{-1/4}$ increases with the increases of ΔT_w. Further, when 'c' increases in the range $0 \le c \le 1$ (i.e., for different shapes) $C_{fy}(Gr)^{-3/4}$ and $Nu(Gr)^{-1/4}$ increases whereas $C_{fx}(Gr)^{-3/4}$ decreases. To be more specific, the percentage of increase in $C_{fy}(Gr)^{-3/4}$ and $Nu(Gr)^{-1/4}$ is about 31% and 11.8%, respectively, when 'c' varies from 0.0 to 0.5, whereas the percentage of decrease in $C_{fx}(Gr)^{-3/4}$ is about 2.6% at $\Delta T_w = 10.0$. As a result there is a reduction in the momentum boundary layer thickness along y- axis and increase in the momentum boundary layer thickness along x-axis, while the thermal boundary layer thickness is reduced as 'c' increases.

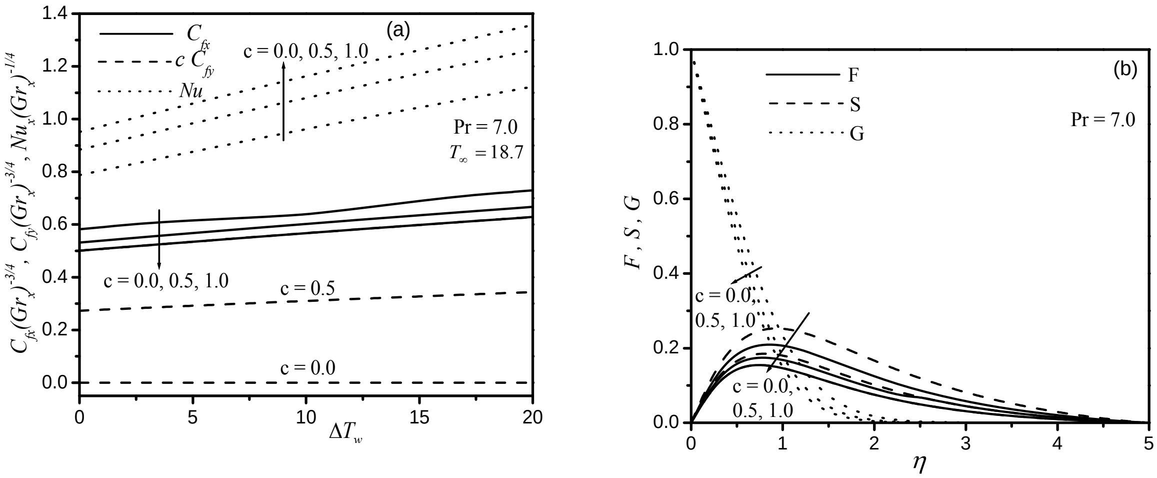


Fig. 4. Variation of (a) skin friction and heat transfer coefficients with ΔT_w (b) velocity and temperature profiles

NOMENCLATURE	
x streamwise coordinate	a, b parameters of principal curvature
y transverse coordinate	F'_w skin friction parameter along x-direction
z normal coordinate	S'_w skin friction parameter along y-direction
f, s dimensionless stream functions	G'_w heat transfer parameter
F,S dimensionless velocity components in x- and y- directions	
T,G dimensional and dimensionless temperatures	*Greek symbols*
u,v,w velocity components in the x-, y- and z-direction, respectively	ν kinematic viscosity
	μ the dynamic viscosity
g acceleration due to gravity	η transformed similarity variable
Pr Prandtl number	α thermal diffusivity
$C_{f\mathrm{x}}$, skin friction coefficient in x- direction	ρ fluid density
$C_{f\mathrm{y}}$ skin friction coefficient in y- direction	
Nu Nusselt number	*Subscripts, Superscript*
c parameter characterizing the nature of the three-dimensional stagnation point	w, ∞ conditions at the wall surface, and at infinity, respectively ,
Gr Grashof number	$'$ differentiation with respect to η

4. CONCLUSIONS

Under the influence of variable viscosity, the steady free convection laminar boundary layer flow at a three dimensional stagnation point has been investigated. From the present analysis, the following observations are noted:

- The effect of 'c' is to increase the skin friction coefficient along y-axis and heat transfer coefficient whereas its effect is to decrease the skin friction coefficient along x-axis.
- Variable viscosity increases both skin friction and heat transfer coefficients which results in the increase of momentum and thermal boundary layer thickness
- The temperature difference between the wall and the fluid has significant effect on both skin friction and heat transfer coefficients.

***Acknowledgment*:** The authors are thankful to PETRF and the Department of Mathematics, PES College of Engineering, Mandya-571 401 for providing excellent facilities for research.

REFERENCES

1. I. Pop, R.S.R. Gorla and M. Rashidi, “The effect of variable viscosity on flow and heat transfer to a continuous moving flat

plate", International Journal of Engineering Science, Vol. 30 No. 1, pp. 1-6, 1992.

2. P. Saikrishnan and S. Roy, "Steady nonsimilar axisymmetric water boundary layers with variable viscosity and Prandtl number", Acta Mechanica, Vol. 157, Nos 1-4, pp. 187-199, 2002.

3. S. Mukhopadhyay, "Effects of radiation and variable fluid viscosity on flow and heat transfer along a symmetric wedge", Journal of Applied Fluid Mechanics, Vol.2, pp.29-34, 2009.

4. G. Poots, "Laminar free convection on the lower stagnation point on an isothermal curved surface, Int. J. Heat Mass Transfer", Vol. 7, 863, 1964.

5. W.H.H. Banks, "Laminar free convection flow at a stagnation point of attachment on an isothermal surface", J. Engng. Math, Vol.8, 45, 1974.

6. S. Sharidan, N. Amin and I. Pop, "G..Jitter free convection flow in the stagnation point of a three-dimensional body", Mech. Res. Comm. Vol.34, pp.115-122, 2007.

7. N. B. Vargaftik, Thermo Physical Properies of Liquids and Gases. John Wiley and Sons, Inc. London, 1975.

8. K. Inouye and A. Tate, "Finite-difference version of quasilinearization applied to boundary layer equations", A.I.A.A.J., Vol.12, pp. 558-560, 1974.