



## A STUDY ON SKEWED MASS DISTRIBUTION

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### Abstract

Civil Engineering structures have to withstand natural environmental forces like wind, earthquake forces and wave forces, along with loads that they are designed to resist. All this environmental forces are random and dynamic in nature. Therefore the response of the structure is also dynamic and that is what causes the unsafe and uncomfortable conditions. Therefore there is always a need for some sort of control of response of structure. This project aims at studying both methods of the Tuned Mass Dampers. It has been well established that Single tuned mass damper (STMD) and Multiple tuned mass damper (MTMD) are effective in reducing the response of the structure. The project aims and study of two devices, the Single Tuned Mass Damper and Multiple Tuned Mass Damper using new control strategy. The tuned mass dampers, consisting of one larger mass block (i.e. one larger tuned mass damper) and one smaller mass block (i.e. one smaller tuned mass damper), referred in this report as the STMD, have been studied to seek for the mass dampers with high effectiveness and robustness for the reduction of the undesirable vibrations of structures under the ground acceleration. Multiple tuned mass dampers (MTMD) consisting of many active tuned mass dampers (TMDs) with uniform distribution of natural frequencies have been proposed to attenuate undesirable oscillations of structures under the ground acceleration

**Key word:** MTMD, STMD, DMF, Parabolic mass, Skewed, Bell shape

### 1. Introduction

Civil Engineering structures have to withstand environmental forces like wind, earthquake forces and wave forces along with loads that they are designed to resist. All this environmental forces are random and dynamic in nature. Therefore the response of the structure is also dynamic and that is what causes the unsafe and uncomfortable conditions. Therefore there is always a need for some sort of control of response of structure. The fact is more important in present times due to following factors:

**1. Increased flexibility:** it is now a necessity and trend to use tall, long or in general more flexible structures. There is also a growing tendency to use lighter and more flexible construction materials. These factors promote the idea of control of vibrations of structure.

**2. Increased safety levels:** As structure becomes more complex, costly and as it serves more critical function, it demands higher safety levels.

**3. Stringent performance requirements:** Structures are required to respond to the forces acting on them within the safety limits. Hence for environmental loads, which are random and dynamic in nature, more stringent safety limits are generally set, which demand for control of vibrations of the structure. Due to the above listed reasons, the concept of structural perception using control systems is not only becoming increasingly popular but it is becoming almost a necessity in modern days. The Tuned Mass Damper is a classical engineering device that is used for vibration control. It consists of mass, a spring and a damper, which is attached to the main structure Fig 1.

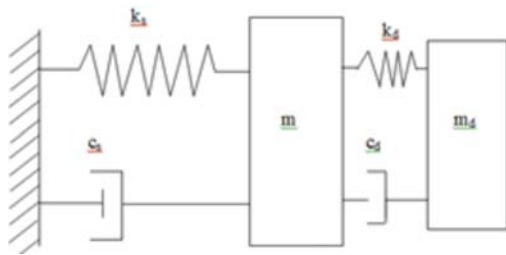


Fig 1 The Single tuned mass damper.

The mechanism of suppressing structural response by attaching tuned mass damper to the structure is to transfer the vibration energy of the structure to the tuned mass damper and to dissipate the energy at the damper of TMD. In order to enlarge the dissipation of energy in TMD, it is essential to tune the natural frequency of TMD to that of structural motion. The TMD has many advantages like compactness, reliability, efficiency and low maintenance cost as compared to other damping devices. Hence, it is widely used in civil engineering structures. Single tuned mass dampers (STMD) have proved to be very sensitive even to the small offset in tuning ratio when it is optimally designed. This is the greatest disadvantage of STMD. This is due to following reasons. Errors in predicting or identifying the natural frequency of the structure and also the error in fabricating a TMD are inevitable to some degree. Some structures have nonlinear properties even in small amplitude range due to contribution of secondary members. Therefore, in practical design the optimum values of parameters of TMD are not chosen. The damping of the TMD is intentionally made higher than the optimal value such that TMD become less sensitive to tuning errors. This results increase the mass of TMD to meet the design requirement. All these uncertainties can be reduced by use of Multiple Tuned Mass Dampers (MTMD). Use of MTMD has been proposed to increase the robustness of the vibration control system to various uncertainties in the structures and/or TMD. The basic configuration of MTMD is the large number of small oscillators whose natural frequencies are distributed around the natural frequency of the controlled mode of structure. It is now well established that an optimal MTMD is more effective and robust than optimal STMD.

## 2. Literature review

Tuned mass dampers (TMD) are widely used to control the vibrations in civil engineering

structures. Although TMDs are effective in reducing the vibrations caused by stationary excitation forces, their performance to suppress seismic response is limited. This inefficiency is due to the fact that TMDs usually need some time interval before it becomes fully effective because they are initially at rest, while the strongest seismic ground motion is often observed at the earlier stage of an earthquake. Another drawback is that TMDs are sensitive to tuning error. Employing more than one tuned mass damper with different dynamic characteristics has then been proposed to further improve the effectiveness and robustness of the TMD. The multiple tuned mass dampers (MTMD) with the distributed natural frequencies were proposed by Xu and Igusa (1991) and also studied by, Abe and Fujino (1994), Abe and Igusa (1995), Bakre and Jangid (2004), Chen and Wu (2001), Gu et al.(2001), Han and Li(2005), Jangid(1995), Joshi and Jangid (1997), Kareem and Kline(1995), Kamiya et al.(1992), Li(2000), Li and Liu(2004), Li and Qu(2000), Lin(2005), Park(2001), Wang(2005), Yau(2004), Yamaguch and Hampornchai(1993). The MTMD is shown to possess better effectiveness and higher robustness in mitigating the oscillations of structures with respect to a single TMD.

### 2.1 Studies of TMD

Likewise, the dual-layer multiple tuned mass dampers, referred to as the DL-MTMD consisting of one larger tuned mass damper and several smaller tuned mass dampers with the total number of tuned mass damper units being the arbitrary integer and with the uniform distribution of natural frequencies have been further proposed by Li (2005) to seek for the mass dampers with high effectiveness and robustness for the reduction of the undesirable vibrations of structures under the ground acceleration. The numerical results indicate that the DL-MTMD can render better effectiveness and higher robustness to the change in the natural frequency tuning (NFT), in comparison with the multiple tuned mass dampers (MTMD) with equal total mass. In fact the DL-MTMD will degenerate into the double tuned mass damper when the total number of the smaller tuned mass damper units in the DL-MTMD is set to be equal to unity. The investigations by Li (2005) have manifested that the DL-MTMD has a little better effectiveness with respect to the DTMD, but they practically reach the same level of robustness to the change in the natural frequency tuning

(NFT). The DTMD consists of one larger mass block (larger tuned mass damper) and one smaller mass block (i.e. smaller tuned mass damper), thus implying that it is significantly simpler to manufacture the DTMD in comparison with the DL-MTMD. With a view to the engineering design and practical applications, it is imperative and of practical interest to carry on further investigations on the DTMD.

Active TMDs can be effective in reducing seismic response because the TMD amplitude can be increased much faster through the use of the actuators. They can also be more robust to tuning errors with the appropriate use of feedback. Therefore, active TMDs have attracted broad research interest and various control algorithms have been developed Yang et al. (1987), Spencer et al. (1994), Chang and Yang (1994). Because of their efficiency and compactness, active TMDs have been successfully designed and installed in full scale Kobori (1991).

Yao (1972) made an attempt to stimulate interest among structural engineers in the application of control theory in the design of civil engineering structures. This has been concluded that much more work is needed in order to apply the concept of structural control to complicated structures such as extremely tall buildings or long bridges subjected to uncertain dynamic loads such as wind and earthquake excitations.

Modern control theories that were developed during the past decade have been successfully applied to the control of the trajectory and motions of space vehicles as well as aeronautical systems. Recently, the control theory has also been applied to reduce the vibration of civil engineering structures Yang (1975). The major difficulty to be encountered is that most civil engineering structures have been very heavy.

Experiments on active control of Seismic Structures have been presented by Chung et al (1998) in which the first phase of a comprehensive experimental study concerning the possible application of active control to structures under seismic excitations is discussed. The experiment consisted of a single degree of freedom model structure, controlled using prestressing tendons connected to the servo hydraulic actuators. An optimal closed loop control scheme using a quadratic performance index was employed to reduce the response of

structure under base motion generated by a large scale seismic simulator. Using a carefully designed, fabricated, and calibrated experimental setup the correlation between the analytical and experimental results was studied. Based on similitude relations, the experimental results obtained for the model structure was extrapolated to the full scale structures are analyzed.

Reinhom et al. (1987) presented a methodology for the shape control of structures undergoing inelastic deformations through the use of an active pulse force system. To avoid the large deformation in structures like tall buildings, long bridges and offshore platforms external forces are applied to the structure through cables, air jets, or other devices in order to ensure that the deformations are kept below the limits set for serviceability at all times.

Yang (1975) investigated the feasibility of optimum active control theory for controlling the motion and vibration of civil engineering structures. It is assumed that the structural system can be discretized, such that the equation of motion can be described by a system of ordinary differential equations. The effectiveness of the control system is measured by a performance index. The optimal control law, which minimizes the performance index, is a linear feedback control. The optimal control forces are obtained by solving a matrix Riccati equation. Moreover, the feasibility of implementing the active control by means of active dampers and servomechanism is considered there.

Abe (1996) is also proposing a rule based on control algorithm for active TMDs. First, perturbation solutions of the linear quadratic regulator (LQR) feedback gains for the active TMD system are derived. Using these solutions interaction of the TMD and the actuator force is discussed in detail. The algorithm consisted of two parts: (1) a variable gain displacement feedback control (2) a variable TMD damping control. The first one is applied when the TMD amplitude is small to make the TMD more effective, and the second one is applied when the TMD amplitude is large to dissipate the energy.

Sarjeet et al (1998) presented a control strategy based on the combination of feed forward and feedback gain controls (an open-closed loop) for the reduction of the displacement response of the shear frame model

of tall buildings to random ground motion which is represented by double filtered white noise.

**3. Methodology**

It was observed that the typical response curve for both constant mass and symmetric mass distributions schemes is a two peak response curve. The two peaks are of unequal heights with the left peak higher than the right one. Den Hartog<sup>44</sup> optimized the single TMD structure system by first equating the height of the two peaks and then adjusting the damping to get a flat response curve. Due to the complexity of MTMD system an analytical solution is not possible. However, the basic idea of attaining equal peak response is adhered to. A mass distribution scheme is used where mass is skewed towards left side to obtain a low and flat peak response. Two different distributions were tried out. These are discussed in details in the following section.

**3.1 Modified parabolic distribution**

The parabolic distribution gave rise to families of response curves that had a higher DMF to the left of the natural frequency of the structure  $\left(\frac{p}{\omega_s} < 1\right)$ . This suggests that instead of having a symmetrical distribution we should use a skewed distribution with more mass towards left. A modified distribution of the form is used.

$$\mu_i = \left\{ \left[ 1 - \left( \frac{n+1}{2} \right)^2 \right] a - \left( i - \frac{n+1}{2} \right)^2 \right\} \gamma(i)^r \quad (3.1)$$

The sign of parameter r would skew the distribution to either side and its magnitude can be used to alter the degree of shift in distribution. The results show that a correct combination of parameters yields a symmetrical and flat peak response. The parameters were varied systematically and preliminary values for each parameter were arrived at by choosing the one which gave minimum value for maximum DMF and a flat response.

From Fig 2, an ‘a’ value of 10 is chosen for further calculations. Fig 3 shows that small damping ratios give rise to large secondary peaks, caused by resonance in the TMD’s one of which gives the maximum response of the structure. The secondary peaks flatten out with increasing damping ratios. This is then the maximum response of the structure. The maximum structural response however increases if the damping ratio is increased too much.

Therefore there exists an optimum value of the damping ratio for an MTMD-Structure which in this case is 0.01. As in previous cases, the optimum frequency bandwidth is found to be close to 0.2 for modified parabolic distribution also Fig 4. A symmetrical and reasonably flat response is obtained at r=2.5. The optimum response is obtained at r=2.5,  $\xi_d=0.012$ ,  $\beta=0.2$ , a= 10 (1% structural damping). The maximum DMF is 10.181 and bandwidth of flat portion is 0.1924. The optimum curve is plotted on linear scale. The total number of TMD’s is taken to be 11 and the total mass is assumed to be one percent.

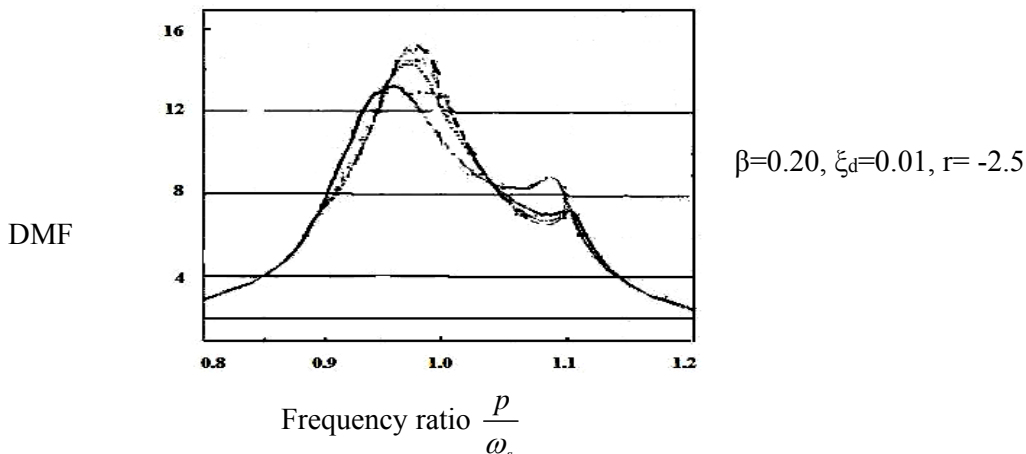


Fig 2 Frequency response curves of structures for different shape factors for modified parabolic distribution while keeping other parameters constant.

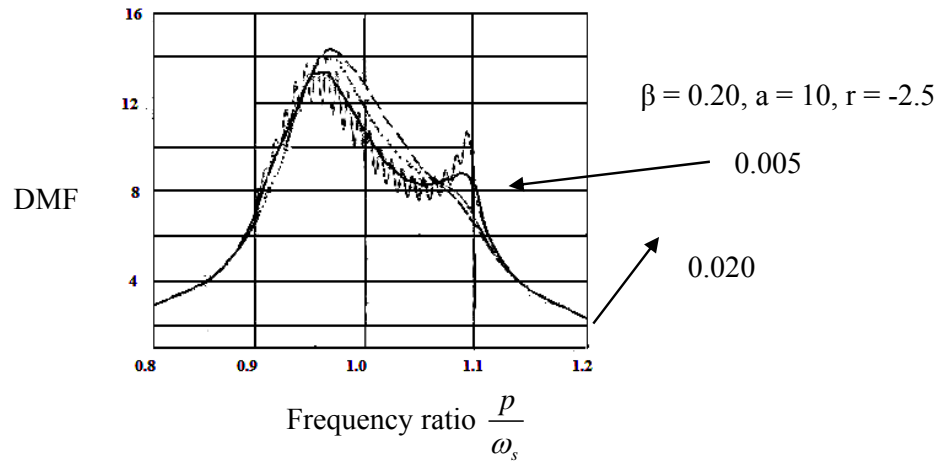


Fig.3 Frequency response curves of structures for different damping ratios factors of modified parabolic distribution while keeping other parameters constant

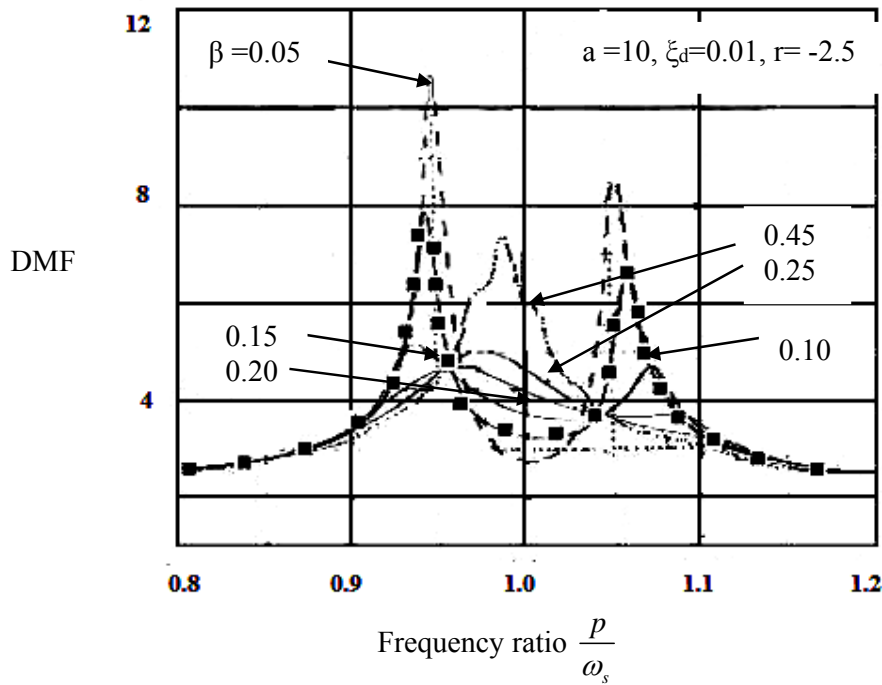


Fig.4 Frequency response curves of structures for different frequency bandwidth for parabolic distribution while keeping other parameters constant.

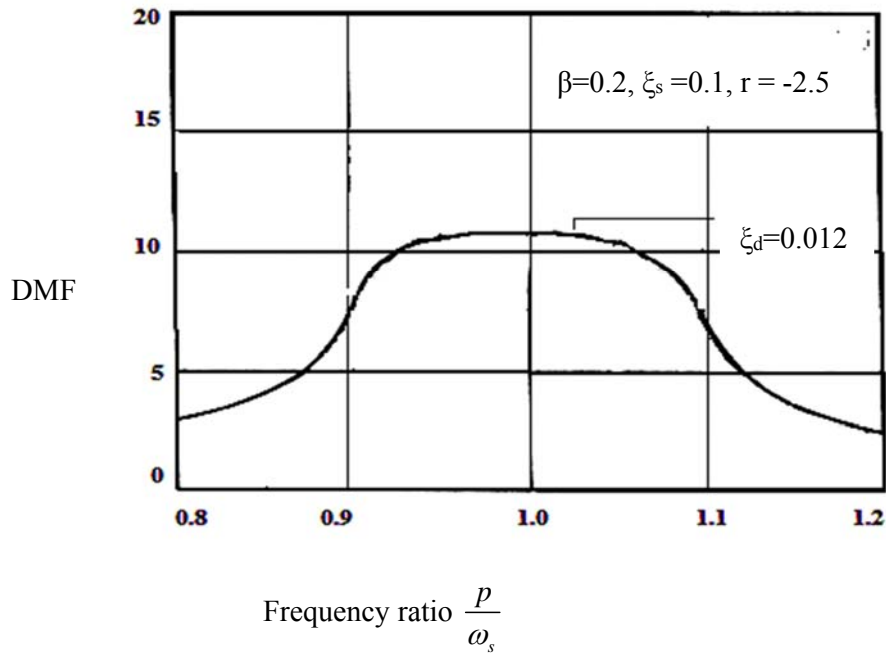


Fig .5 Optimum parameter curve for modified parabolic distribution.

### 3.2 Bell shaped distribution

The modified parabolic distribution works well to give a reduced and a symmetrical distribution. However, a completely flat peak response was not being attained. This means that the idea of skewed mass distribution to achieve a symmetrical distribution holds. A better distribution is needed to get a perfectly flat response curve. It can be seen that typical response of these systems is bell shaped. So it was decided to use a bell shaped distribution that is similar to the response of the system. The option of skewing

the mass towards left is maintained.

Because of the above considerations a bell shaped distribution of the form is used.

$$\mu_i = \left[ \frac{a^3}{a^2 + \left(i - \frac{n+1}{2}\right)^4} \right] \gamma(i)^{-n} \quad (3.2)$$

The following figure shows the mass distribution for  $a=60$ .

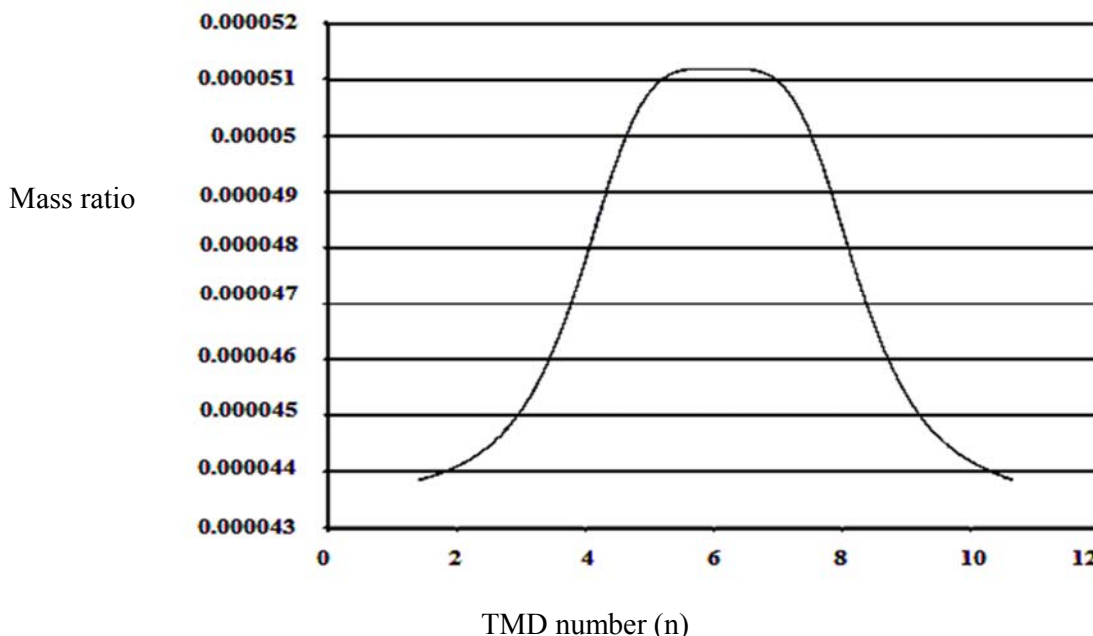


Fig 6 Bell shaped distribution for a = 60

The sign of parameter  $n$  would skew the distribution to either side and its magnitude can be used to alter the degree of shift in distribution. The results show that a correct combination of parameters yields a symmetrical and flat peak response. The parameters were varied systematically and preliminary values for each parameter were arrived at by choosing the one which gave minimum value for maximum DMF and a flat response. Total mass ratio is again assumed to be one percent. Fig. 7 shows that a reasonably low and flat response is obtained at about  $a=60$ . Fig 8 suggests a value

of 0.015 for  $\xi_d$ . This is one of the possibilities although other solutions might exist. The best value for each parameter obtained from the graphs is used in every successive one. Therefore varying local skewing parameter gives us the local optima. A symmetrical and reasonably flat response is obtained at  $n=3.2$ . The optimum response Fig. 11 is obtained at  $\xi_d = 0.015$ ,  $\beta=0.2$ ,  $a=60$  (1% structural damping). The maximum DMF is 9.352 and bandwidth of flat portion is .2061. The optimum curve is plotted on linear scale.

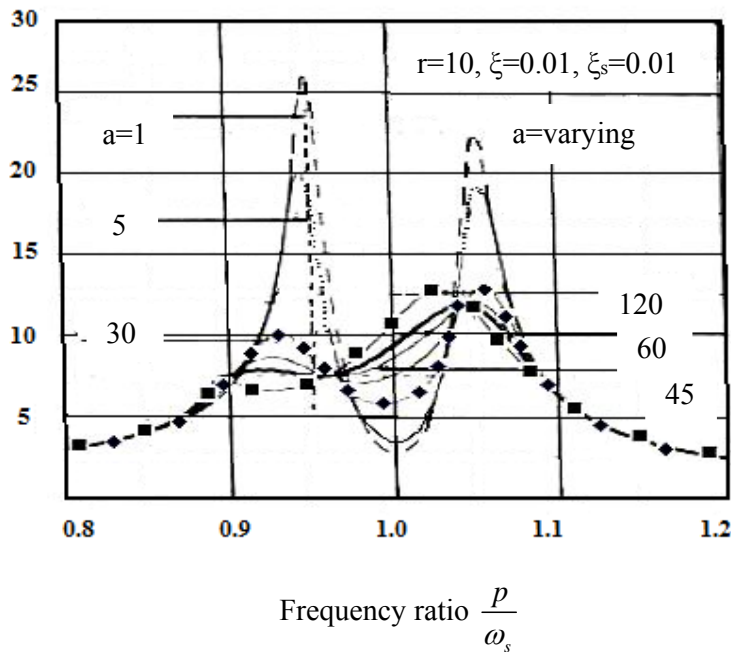


Fig.7 Frequency response curves of structures for different shape factors for bell shaped distribution while keeping other parameters constant.

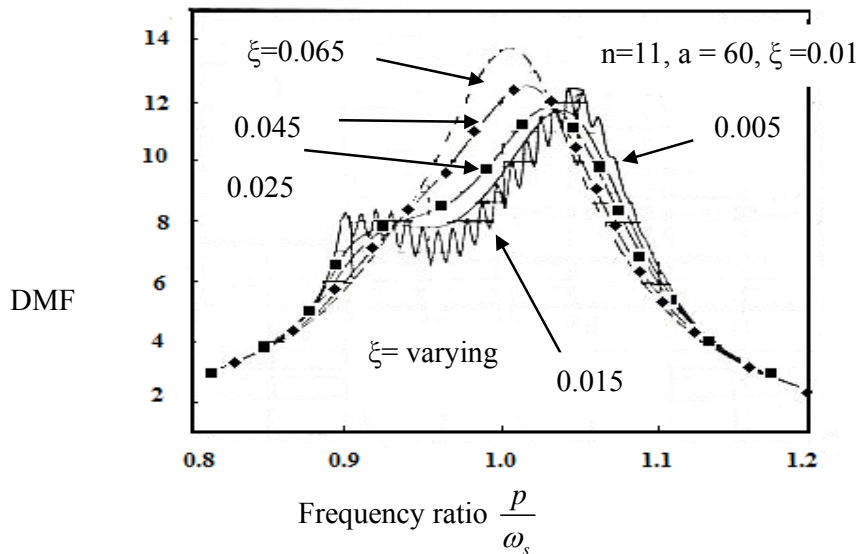


Fig. 8 Frequency response curves of structures for different damping ratios factors of bell shaped distribution while keeping other parameters constant.

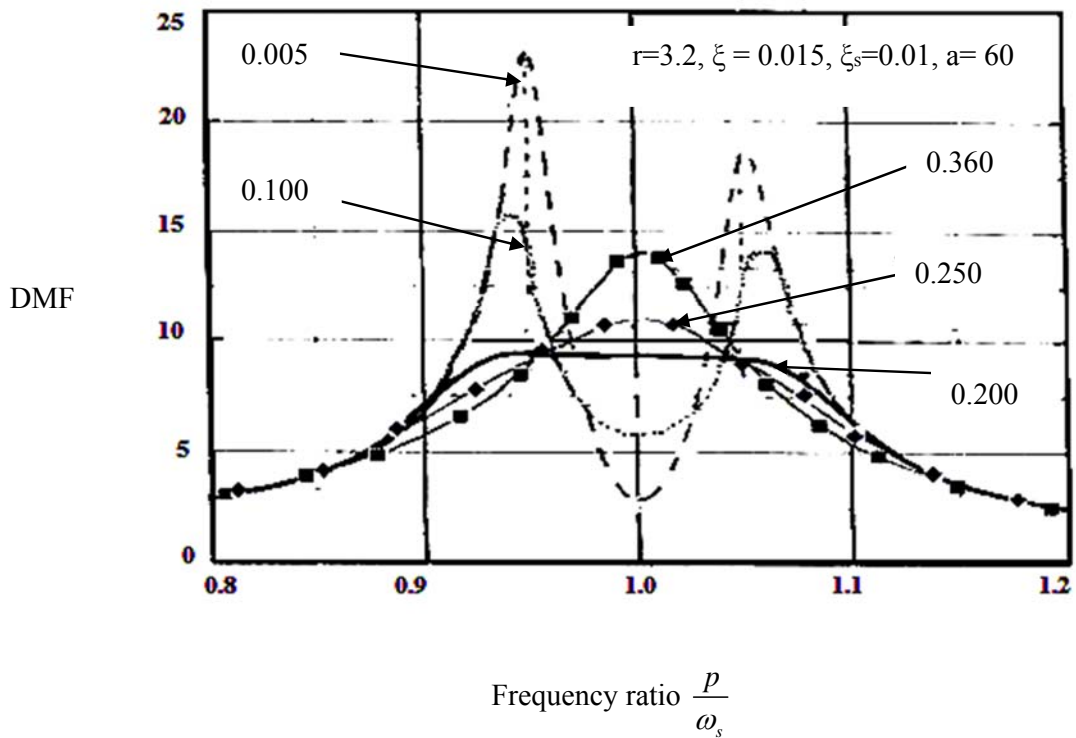


Fig. 9 Frequency response curves of structures for different frequency bandwidth for bell shaped distribution while keeping other parameters constant

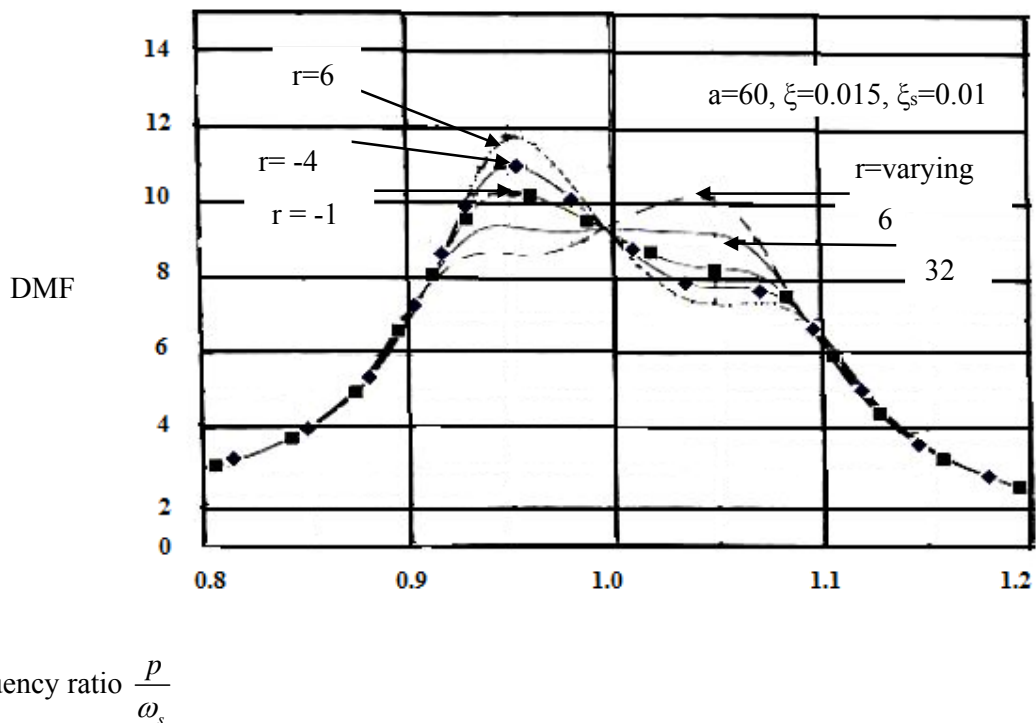


Fig. 10 Response factors for varying n values while keeping other parameters constant

The best value of each parameter from the above graphs is taken and hence after varying the parameter 'r' a perfectly symmetrical and flat response curve is obtained with a reduced DMF as shown in the figure below.



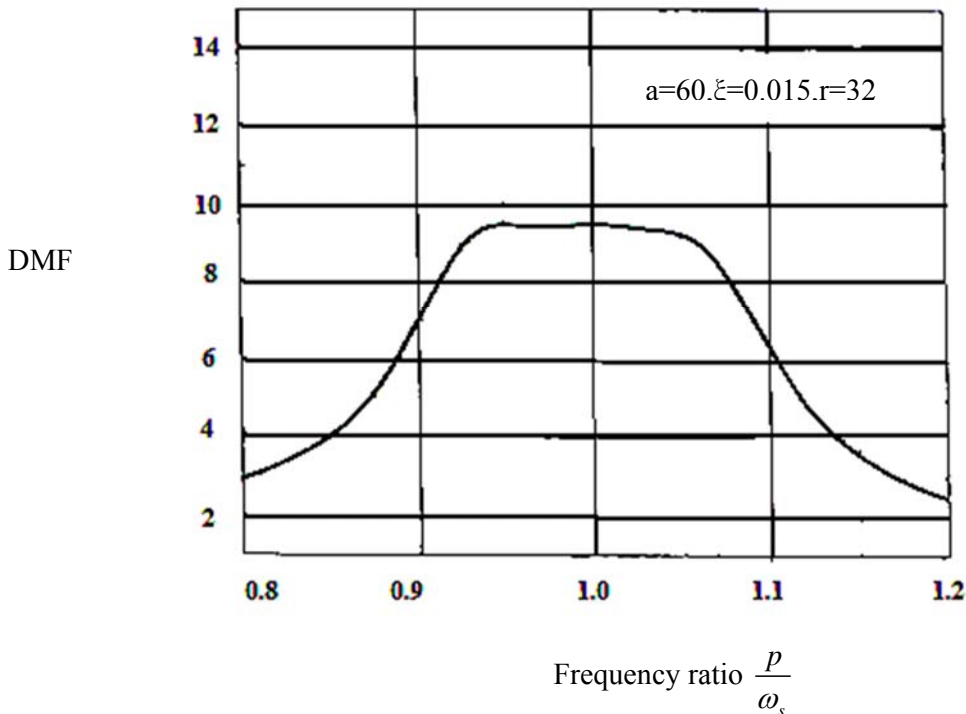


Fig. 11 Optimum parameter curve for bell shaped distribution

The maximum DMF obtained is 9.352 which is considerably lower than the value obtained for parabolic distribution (10.212) and modified parabolic distribution (10.181). Also the bandwidth of flat portion is 0.2061.

### Conclusions

According to the above results, it can be said that the frequency range of MTMD is the most important design parameter in the sense that main peak in the response curve is dominated by the frequency range of MTMD. If the frequency range is appropriately selected, the response curve of the structure can be flattened in the wide range of the resonant region. Once the frequency range of an MTMD is determined only problem that remains is to suppress the secondary peaks which can be done by choosing appropriate values of the number and damping of TMD's. This can be done in two ways; (a) smaller number of TMD with large damping ratios and (b) Large number of TMD's with small damping ratios. If fewer TMD's are used, spacing of distributed frequency becomes large which leads to large secondary peaks which are then reduced by using increasing the damping ratios. On the other hand if number of TMD's used is large the damping ratio should be made smaller because the secondary peaks cannot be significant owing to interaction of closely spaced TMD.

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