PROBABILISTIC STRESS ANALYSIS OF CYLINDRICAL PRESSURE VESSEL UNDER INTERNAL PRESSURE USING MONTE CARLO SIMULATION METHOD

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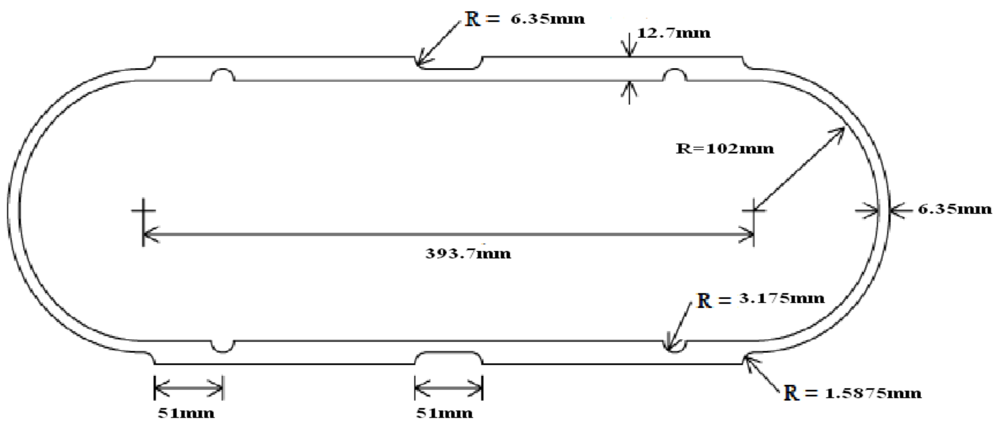
Abstract
Probabilistic stress analysis of cylindrical pressure vessel under internal pressure is performed using FEA. The pressure vessel was modeled as an axisymmetric shell with symmetric boundary condition along the two unloaded edges. The loaded edges were treated as clamped in this study. Monte Carlo simulation technique is used to estimate the probability of failure of the cylindrical pressure vessel by considering uncertainties. Here variations in material properties and loads are assumed to follow the Normal distribution with 1 to 5% variations this analysis has been carried out by using PDS facility of ANSYS software. The ANSYS Probabilistic Design system (PDS) analyzes a component or a system involving uncertain input parameters. The probability of failure is calculated by applying the limit state equation. From that, the reliability of the pressure vessel component is estimated. This result is used to make a decision to produce a component for required reliability.
Keywords: Probabilistic design systems, composite cylinder, Monte Carlo Simulation, ANSYS PDS software, Normal distribution, Reliability.

1. INTRODUCTION

Pressure vessels are used in a number of industries; for example, the power generation industry for fossil and nuclear power, the petrochemical industry for storing and processing crude petroleum oil in tank farms as well as storing gasoline in service stations, and the chemical industry[1]. The inside pressure is usually higher than the outside, except for some isolated situations. The fluid inside the vessel may undergo a change in state as in the case of steam boilers or may combine with other reagents as in the case of a chemical reactor. Pressure vessels often have a combination of high pressures together with high temperatures and in some cases flammable fluids or highly radioactive materials. Because of such hazards, it is imperative that the design be such that no leakage can occur. In the design of pressure, vessels Uncertainties exist in dimension, applied loads, boundary conditions, and material properties from manufacturing. Therefore, it is very important to qualify uncertainty and to define approximate deviations in manufacturing, which can be probabilistic or statistical [2].

1.1 Stresses in pressure vessel under internal pressure.

Cylindrical pressures have been extensively used in industries like power, chemical etc. They are subjected to stresses due to storage of pressurized fluids inside the vessels



a). Longitudinal stress (axial) σ_L

(b). Hoop stress σ_h

1.2 Probabilistic design

Uncertainties in design have been accounted for traditionally by means of a safety margin or "safety factor." The selection of a safety factor is usually based on experience and engineering judgment. In recent years, the experience and judgment relied upon for selection of a safety factor has been augmented by analytical methods which permit probabilistic treatment of the known load and strength variables affecting a design problem [3].

The probability that failure will occur is defined as the probability that an induced stress (S_i) will exceed the allowable stress (S_a) for any values of S_i. and S_a that exist simultaneously. The analytical process used to calculate this probability is known as probabilistic design [4],[8]. The interaction of the induced stress distribution and the allowable stress distribution is illustrated in Fig. 1. Since these are frequency distributions, the probability of failure is a function of the area of overlap of the two curves. When the distributions are defined, probability calculus can be used to calculate the area of overlap of the curves

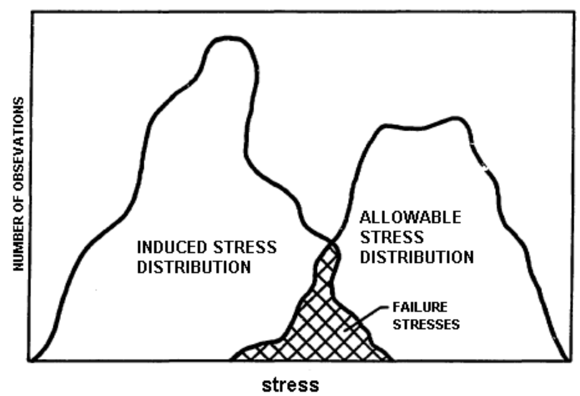


Fig1. Failure stress resulting from overlap of two stress distributions.

For failure as defined herein, the probability of failure (Q) is the probability that an induced stress (S_i) will exceed the allowable stress (S_a) for any values of the induced stress and allowable stress that exist simultaneously.

$$Q = P(S_i \geq S_a) \quad (1.1)$$

When inequality operations are applied to Equation 1.1, the probability of failure can also be stated as

$$Q=P(S_a-S_i\leq 0) \quad (1.2)$$

Frankel, E. G[5] intended to provide a practical introduction to reliability analysis and risk assessment that can be used by professionals in engineering, planning, management, and economics to improve the design, operation, and risk assessment of systems of interest. It includes concepts of probability and statistics, reliability function, reliability of series and parallel systems, fault trees analysis, multivariate probability distribution and stochastic processes, testing for Markov properties, the generalized failure process for non-maintained systems, analysis of maintained system, strategies for repair policies, effects of component interaction, application of fault tree and other network techniques.

Bucher[6] suggested an iterative Monte Carlo simulation procedure, which utilizes results from simulation to adapt the importance sampling density. He also reduced the statistical error of the estimated failure probability. His method is especially suitable for system reliability analysis since multiple failure modes need not be treated separately.

The first part of this study is focused on the deterministic analysis of cylindrical pressure vessel under internal pressure. The deterministic induced stress calculated using ANSYS software is validated and compared with the theoretical calculation The second part of this study examines the reliability of the cylindrical pressure vessel under pressure. The Monte Carlo Simulation technique is used to find the Maximum induced stress for 500 simulation. Monte Carlo Simulation (MCS) technique is used to estimate the probability of failure of the cylindrical pressure vessel by considering uncertainties.

Then probability of failure is calculated by applying the limit state equation. From that reliability of that component is estimated. This result is useful to make decision to produce component for required reliability.

2. DETERMINISTIC FINITE ELEMENT ANALYSIS

2.1 General Procedure For Deterministic Stress Analysis

- Define Parametric Variables.
- Building the Parametric model.
- Meshing.
- Apply Boundary Conditions.
- Solving and post processing of the finite element model

2.2 *Definition of Parametric Variables.*

Table .2.1 Definition of random input variables

S.No	Random input variables	Distribution	Standard deviation
1	Length(L) = 393.7mm	Normal	2%
2	Radius(r)=102mm	Normal	1%
3	Thickness(t)=6.35mm	Normal	1%
4	Fillet radius(R_1)=6.35mm	Normal	1%
5	End fillet radius(R_2)=1.5875mm	Normal	1%
6	Width of the groove(w)=51mm	Normal	1%
7	Radius of the inner groove(R_3)=3.175mm	Normal	1%
8	Internal pressure (p)=1700N/mm2	Normal	5%
9	Poisson ratio(μ)=0.28	Normal	1%
10	Young's modulus(E)=$1.20X10^5 N/mm^2$	Normal	1%

The geometric and material properties of cylindrical pressure vessel are given in Table 2.1, where 'L' is the length of the cylindrical portion, 'r' is radius of the spherical end of the cylinder, 't' is the thickness of the spherical end, 'R_1' is the fillet radius at the middle, 'R_2' is the fillet radius at the junction of the cylindrical and spherical end, 'W' is the width of the groove at the middle, 'R_3' is fillet radius at the inner side of the pressure vessel, 'p' is the internal pressure applied at the inner surface of the pressure vessel, 'μ' is the Poisson ratio and 'E' is the young's modulus of the pressure vessel material.

Normal distribution was assumed for all random variable scatters. It is essential to define all the random variables including the random output variable (maximum von-mises stress) for deterministic analysis before building the model, because it will be useful for probabilistic analysis.

2.3 Building the Parametric model.

This cylindrical pressure vessel model exhibits symmetry about the z-axis of rotation, or in other words it is a solid of revolution. The boundary and loading conditions are also symmetric to the axis of rotation. Material properties used in this study are isotropic. In this study the analysis was performed using a 2-D axisymmetric model of the cylindrical pressure vessel.

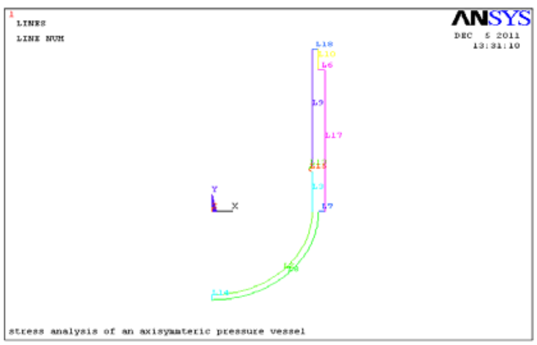


Figure 2.1 shows the axisymmetric model

2.4 *Meshing of axisymmetric model*

The parametrically defined axisymmetric model is converted in to finite element model by meshing. The plane 82 element is used to mesh the axisymmetric model.

2.5 *Apply Boundary Conditions*

The symmetric boundary condition is applied at the upper edge of the axisymmetric model. Finally, pressure can be applied on all lines that make up the inner surface of the pressure vessel

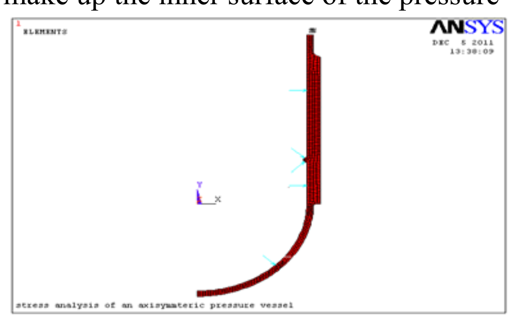


Figure 2.2 shows the boundary condition applied to the finite element model

2.6 Solving and post processing of the finite element model

The finite element model is solved by choosing the analysis type as static to find von-mises stress distribution in the cylindrical pressure vessel. This maximum von-mises stress represents the random output parameter variable. This random out variable is compare with the allowable stress of the pressure vessel material. For model validation purposes, the stresses in the vessel walls away from any notches can be estimated using the thin-walled pressure vessel equations.

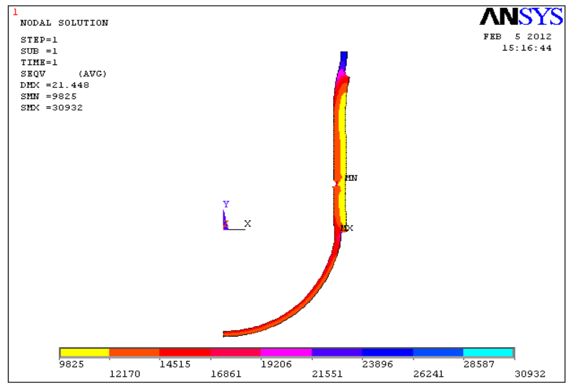


Figure 2.3 shows the von mises stress of the finite element model

Table .2.2 comparison of results

ANSYS Result	Maximum induced stress=30932 N/mm^2
Theoretical result	Hoop stress=27307 N/mm^2

3. PROBABILISTIC FINITE ELEMENT ANALYSIS

The probabilistic finite element analysis of cylindrical pressure vessel under internal pressure was performed by mean of ANSYS/PDS. The PDS is based on the ANSYS parametric design language, which allows users to parametrically build a finite element model, solve it, obtain results and extract characteristics results parameters such as the maximum von-mises stress for example. The PDS includes MCS and RSM[7].

In this work MCS was used to execute the probabilistic analysis of cylindrical pressure vessel under internal pressure. The mechanical properties of cylindrical pressure vessel including young's modulus and Poisson's ratio, and the dimensions of cylindrical pressure vessel are taken as random input parameters and the maximum von-mises stress was taken as random output response. The statistical characteristics for them including mean, coefficient of variation (COV) and distribution type are given in Table 2.1. The uncertainties of mechanical properties and physical dimensions are influenced significantly by the manufacturing of cylindrical pressure vessel. In general, the statistical characteristics are achieved based on the extensive data collection and data analysis. However, in the absence of sufficient and good quality data, professional expertise has to be employed. In this work, the variables values estimated based on tests and engineering judgment are used.

For the MCS, the direct Sampling[9] is selected due to that this technique avoids repeating samples that have been evaluated, and also forces the tails of a distribution to participate in the sampling process. In this work, 500 direct sampling loops are run which are sufficient to obtain converged outputs.

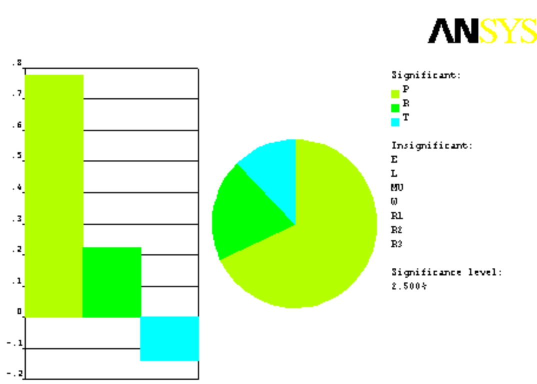


Figure 2.4 Sensitivity plot for von-mises stress

The evaluation of the probabilistic sensitivities is based on the correlation coefficients between all random input variables and a particular random output parameter. Either Spearman rank order correlation coefficients or Pearson linear correlation coefficients may be used based on user's specifications. To plot the sensitivities of a certain random output parameter, the random input variables are separated into two groups[10], those that are significant (important) and those that are insignificant (not important) for the random output parameter. The sensitivity plots will only include the significant random input variables.

Using the limit-state function formulation in Eq. (1.2), a direct Monte Carlo simulation was performed. The allowable stress for the cylindrical pressure vessel is 36000N/mm^2.Specimen failure due to internal pressure was detected by applying the limit state function. The probability of failure is then defined as

$$P_f = \frac{N_f}{N} \qquad (3.1)$$

Where N is total number of simulations and is the number of failures. The Coefficient of variation of failure probability found as

$$COV(P_f) = \frac{\sqrt{\frac{(1-P_f)P_f}{N}}}{P_f} \qquad (3.2)$$

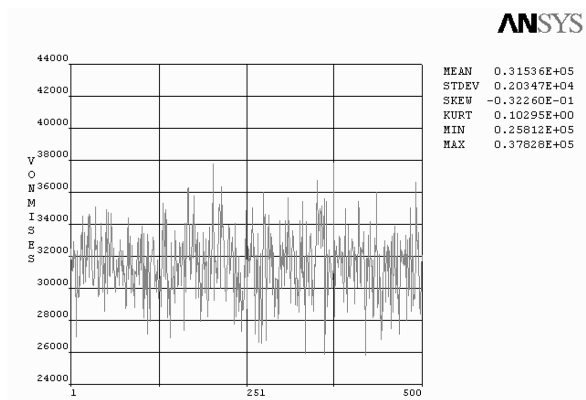


Figure 2.5 Von-mises stress for 500 samples

The 500 numbers of simulation cycles were run and 8 instances were found failure. This resulted in a probability of failure of 0.016 for this specimen based on the assumed distribution, mean and scatters for the applied pressure with the final COV = 3.50%.then by using the equation (3.3) ,we can find the reliability of the cylindrical pressure vessel under internal pressure.

$$\text{Reliability} = (1 - p_f) \times 100 \qquad (3.3)$$

Therefore reliability of the cylindrical pressure vessel under internal pressure was estimated as

98.4 % .this results is useful to make decision to produce component for required reliability.

4. CONCLUSIONS

The reliability of cylindrical pressure vessel under internal pressure was investigated by considering the uncertainties in the input variables using FEA software .this result is useful to make decision to produced component for required reliability. The variations in random input variables are included and corresponding variations in the output parameters were noted .The sensitivity of the induced stress with respect to material properties and geometrical parameters were shown in the sensitivity plot .This results indicate applied pressure, radius and thickness have the greatest influences on induced stress (von mises). The influence of the remaining input parameter was found to insignificant.

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