FFT BASED CLOSED LOOP LINEAR SYSTEM RESPONSE WITH & WITHOUT DELAY ELEMENTS

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Abstract
This paper discusses the application of FFT for closed loop system response of a linear system with and without delay elements and its frequency response. The method requires only closed-loop step response data for system design. The information in time domain of the data is used to estimate the system model, and that in frequency domain is used to confirm the stability of the linear part of the system. Fast Fourier transform (FFT) is measured input and output responses without disconnecting the controller from the loop, which significantly reduces the time required for the auto tuning test. For linear system it has been shown that the plant system in both time and frequency domain can be obtained with the data reported with improved accuracy to estimate process transfer function models. In this approach a delay is connected in series with a system to observe the system and impulse response. Other identification methods like difference equation, Z-Transform, Laplace transform have been verified, using fast Fourier transform (FFT),the execution time of system is fast when compared to above transforms.
Index Terms: closed loop system, Fast Fourier transforms, Laplace transform, Z-Transform.

I. INTRODUCTION

A control system with feedback loop is called “closed loop control system”. In other words, the control system which uses its feedback signal to generate output is called” closed loop control system”. In these control systems, the input is controlled by the feedback signal from input so that it can correct the errors occurred. Closed loop control systems are two way signal flow systems. Feedback means, some part of output is taken and connected it to the input of the system to maintain the stability of the control system. By providing a feedback loop, we can convert any open loop control system into closed loop system. The feedback loop provides the automatic correction of the input signal based on the output requirement.

By comparing the generated output with the actual condition, the closed loop system maintains and achieves the desired output. If the produced output is deviated from decided (actual) output, the closed loop control system generates an error signal and the error signal is fed to the input of the signal.

So by adding the error signal to the input, the generated output of the next loop will be corrected. So these are also called as automatic control systems. Closed loop systems are less prone to external disturbances

A. Fast Fourier Transform (FFT):

A fast Fourier transform (FFT)[2] algorithm computes the discrete Fourier transform(DFT) of a sequence, or its inverse. Fourier analysis converts a signal from its original domain (often time or space) to a representation in the frequency domain and vice versa. An FFT rapidly computes such transformations by factorizing the DFT matrix into a product of sparse (mostly zero) factors. As result, it manages to reduce the complexity of computing the DFT from O(n2), which arises if one simply applies the definition of DFT, to O(n log n) where n is the data size.

Fast Fourier transforms are widely used for many applications in engineering, science, and mathematics. The basic ideas were popularized in 1965, but some algorithms had been derived as early as 1805. In 1994, Gilbert Strang described the FFT as "the most important numerical algorithm of our lifetime" and it was included in Top 10 Algorithms of 20th Century by the IEEE journal computing in Science & Engineering.

B. Definition and speed:

An FFT computes the DFT and produces exactly the same result as evaluating the DFT definition directly; the most important difference is that an FFT is much faster[3]. (In the presence of round-off error, many FFT algorithms are also much more accurate than evaluating the DFT definition directly, as discussed below.)

Let x(0), x(N−1) be complex numbers. The DFT is defined by the formula

$$X_k = \sum_{n=0}^{N-1} x_n \, e^{-i2\pi kn/N}$$

k = 0, 1, 2,………..,N – (1)

Evaluating this definition directly requires O(N2) operations: there are N outputs Xk, and each output requires a sum of N terms. An FFT is any method to compute the same results in O(N log N) operations. All known FFT algorithms require Θ(N log N) operations, although there is no known proof that a lower complexity score is impossible.

To illustrate the savings of an FFT, consider the count of complex multiplications and additions. Evaluating the DFT's sums directly involves N2 complex multiplications and N(N−1) complex additions, of which O(N) operations can be saved by eliminating trivial operations such as multiplications by 1. The radix-2 Cooley–Tukey algorithm, for N a power of 2, can compute the same result with only (N/2)log2(N) complex multiplications (again, ignoring simplifications of multiplications by 1 and similar) and N log2(N) complex additions..

C. Applications:

FFT's importance derives from the fact that in signal processing and image processing it has made working in frequency domain equally computationally feasible as working in temporal or spatial domain. Some of the important applications of FFT include,

- Fast large integer and polynomial multiplication
- Filtering algorithms
- Fast algorithms for discrete cosine or sine transforms (example, Fast DCT used for JPEG, MP3/MPEG encoding)
- Fast Chebyshev approximation
- Fast discrete Hartley transform Submit your manuscript electronically for review.

D. Comparison between FFT and Convolution summation:

FFT:

A fast Fourier transform (FFT) algorithm computes the discrete Fourier transform (DFT) of a sequence, or its inverse. The output of FFT is more accurate compared to convolution summation method. The time taken to execute the system is less compared to convolution summation method

Convolution summation:

In mathematics convolution is a mathematical operation on two functions (f and g); it produces a third function. The output of convolution summation is less accurate compared to FFT.The time taken to execute the system is more compared to FFT method.

E. Frequency Response

The frequency response of a system is defined as the steady-state response of the system to a sinusoidal input signal[5]. The sinusoid is a unique input signal, and the resulting output signal for a linear system, as well as signals throughout the system, is sinusoidal in the steady-state; it differs from the input waveform only in amplitude and phase.

II. CLOSED LOOP SYSTEM RESPONSE USING CONVOLUTION SUMMATION METHOD

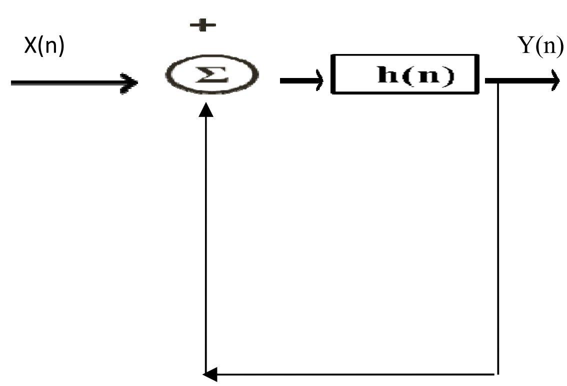

Fig:1

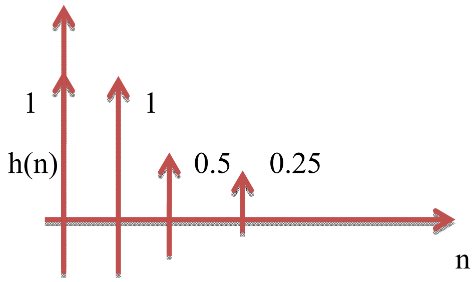

fig:2

$$y(n) = x(n) * h(n) \tag{2}$$

$$y(n) = \sum x(k) * h(n-k) \tag{3}$$

y(n) is the output
x(n) is the input
h(n) is the impulse

Response by using convolution summation:

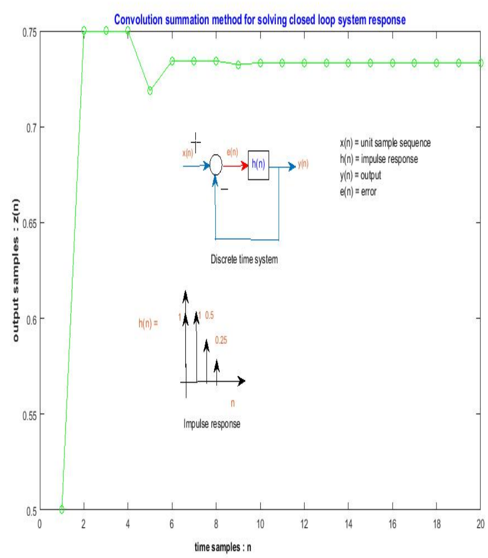

Fig:3

III. CLOSED LOOP SYSTEM RESPONSE USING FFT

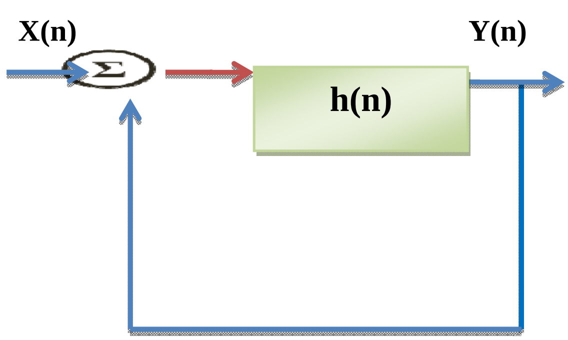

Fig:4

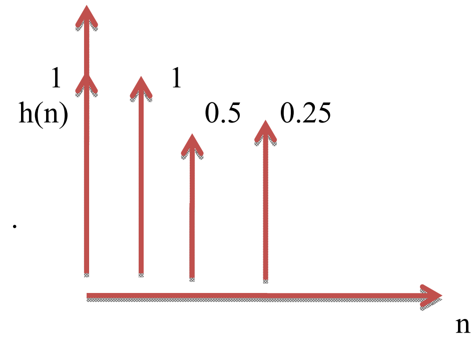

Fig:5

$$X(K) \rightarrow FFT\{\, x(n)\,\} \tag{4}$$

$$H(K) \rightarrow FFT\{\, h(n)\} \tag{5}$$

$$Y(K) = X(K) * H(K) \tag{6}$$

$$y(n) = IFFT\{\, Y(k)\,\} \tag{7}$$

$$y(n) = y(n)/2 \tag{8}$$

Response by using FFT:

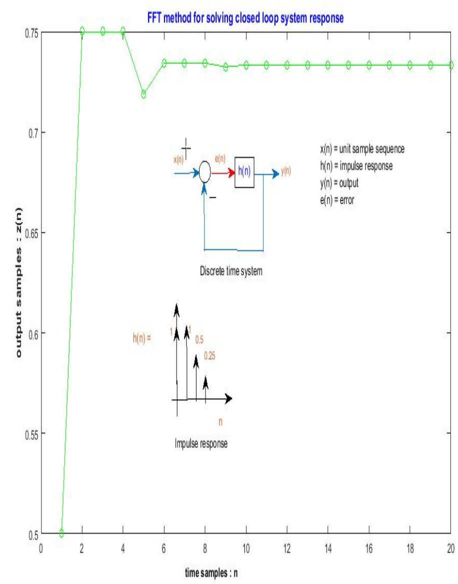

Fig:6

IV. SYSTEM WITH ZOH AND WITHOUT ZOH

The zero-order hold (ZOH) is a mathematical model of the practical signal reconstruction done by a conventional digital-to- analog converter (DAC)[6]. That is, it describes the effect of converting a discrete-time signal to a continuous-time signal by holding each sample value for one sample interval

System considered for with zoh and without zoh:

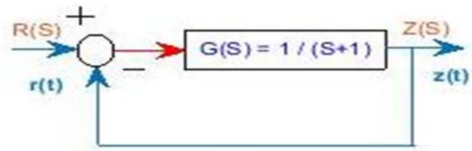

Fig:7

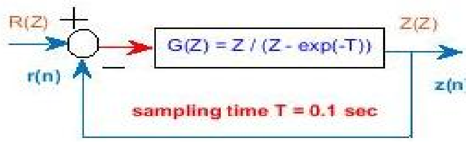

Fig:8

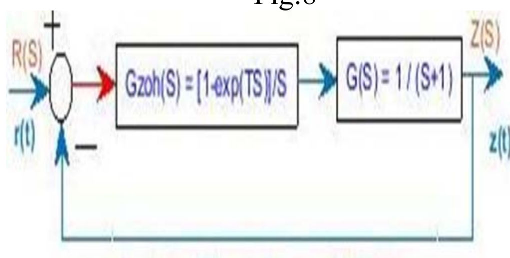

Fig:9

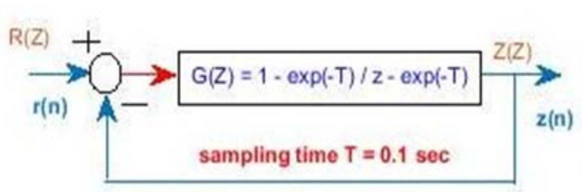

Fig:10

Zoh using convolution summation method:

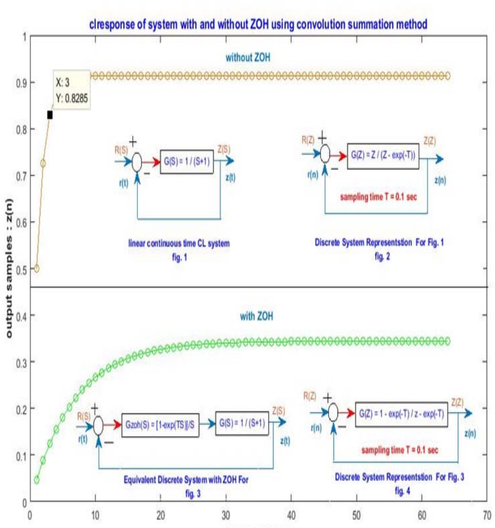

Fig:11

Graph showing time response of first order closed loop system by with and without ZOH by FFT METHOD:

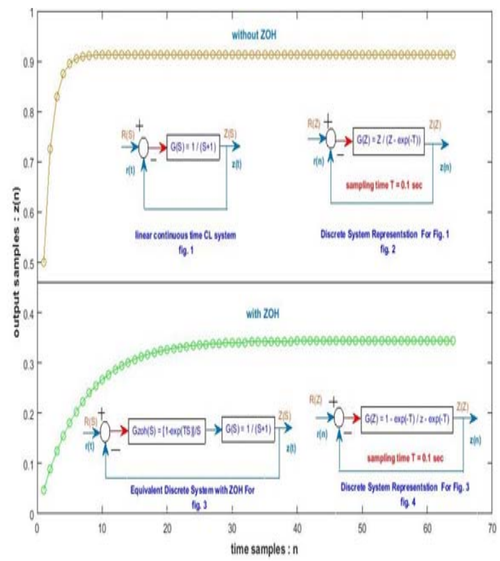

Fig:12

Block Diagram considered for DELAY SYSTEM:

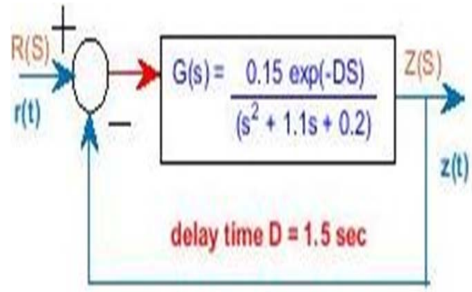

Fig:13

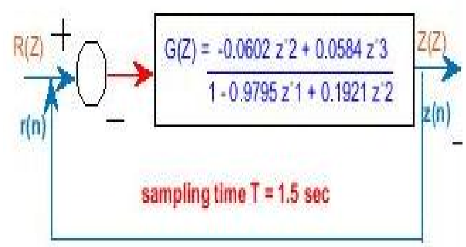

Fig:14

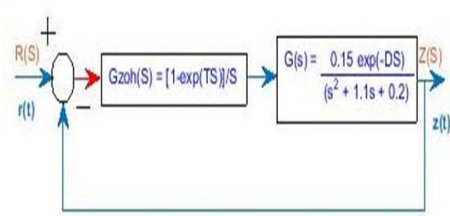

Fig:15

Delay response graph:

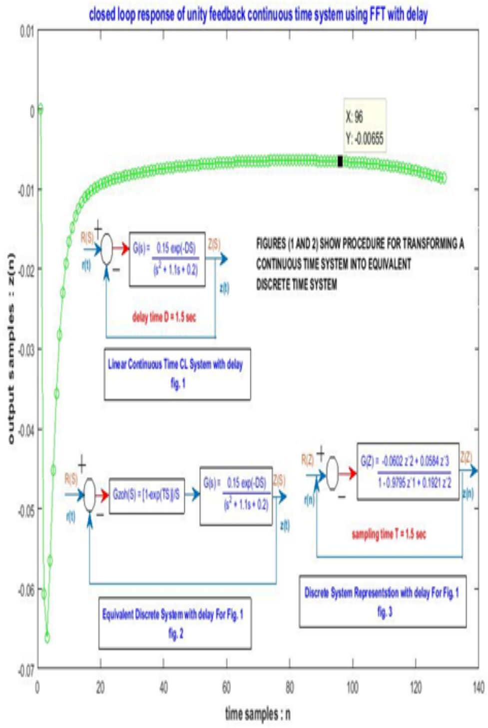

Fig:16

Advantages of Frequency Response:

- Frequency response (mathematical modeling) can be obtained directly by experimental approaches.
- Easy to analyze effects of the system with sinusoidal signals.
- Convenient to measure system sensitivity to noise and parameter variations

V. RESULTS

Time period comparison table for two methods

CONVOLUTION SUMMATION	Fast Fourier Transform
• Closed loop response time response is 0.2028secs	• Closed loop response time response is 0.0156sec
• With ZOH is 34.538sec for 512 input samples	• With ZOH is 0.0624sec for 512 input samples
• Without ZOH is 42.8379sec for 512 input samples	• Without ZOH is 0.1624sec for 512 samples
• Time response with delay is 0.0312sec for 128 samples	• Time response with delay is 0.0156sec for 128 samples

Table: I

VI. CONCLUSION

After comparing the results that are obtained we can see that FFT method is faster when compared to convolution summation method. As we observed FFT method is much faster than Difference equation method.FFT technique is the best method for closed-loop step response data for designing any controller. The frequency domain is used to confirm the stability of the linear part of the system. The output plots have been shown in the results and the results have been compared.

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