IMPLEMENTATION OF AUTO CORRELATION FUNCTION AND WEINER KHINCHIN RELATION A NOVEL APPROACH

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Abstract
The Cross-Correlation theorem relates the Fourier transform of the cross-correlation function of two signals with the Fourier transforms of the individual signals. Let x(t) and y(t) be two integrable signals, 0 everywhere outside t€[0,T], with values in[-1,1]. The Wiener-Khinchin theorem is the special case where x(t) = y(t).
Index Terms: Cross Correlation, Weiner Kinchin relation, Power spectral density.

I.INTRODUCTION

Cosider a random process x(t) (a random variable that evolves in time)with the autocorrelation function

$$C(\tau)=\langle x(t)x(t+\tau)\rangle$$

X is typically thought of as voltage and the terminology stems from this identification but in general it can be any random variable of interest. The brackets denote averaging over an ensemble of realizations of the random variable,e.g., many different traces of the voltage as a function of time. We assume that the process is (weakly)stationary, i.e., the mean value <x(t)> is independent of time and the correlation function only depends on the difference of the time arguments and is independent of t in the equation above. From a practical point of view this is assumed to hold in steady state if the dynamics underlying the process is time translationally invariant.

II. WEINER KINCHIN RELATION PROOF

We will assume that the Fourier transform of C(τ) defined by

$$\hat{C}(\omega) = \int_{-\infty}^{\infty} d\tau\, e^{-j\omega\tau} C(\tau) d\tau$$

Define the truncated Fourier transform of a realization of the random process x(t) over an interval [-T/2,T/2] by

$$\hat{x}_T(\omega) = \int_{-\frac{T}{2}}^{\frac{T}{2}} dt x(t)\, e^{-j\omega t}$$

Since x(t)is a random variable so is $\hat{x}_T(w)$. We define the truncated spectral power density, $S_T(w)$ by

$$S_T(\omega) = \frac{1}{T}\langle|\hat{x}_T(\omega)|\rangle^2$$

The spectral power density of the random process, x(t) is defined by

$$S(w) = \lim_{T\to\infty} S_T(\omega) = \lim_{T\to\infty} \frac{1}{T}\langle|\hat{x}_T(\omega)|\rangle^2$$

The Wiener-Khinchen theorem states (a) that limit in Equation (4) exists and (b) the spectral power density is the Fourier transform of the autocorrelation function,

$$S(\omega) = \int_{-\infty}^{\infty} d\tau\, e^{-jw\tau} C(\tau) +$$

Proof:
Consider

$$\langle|\hat{x}_T(\omega)|\rangle^2 = \int_{-\frac{T}{2}}^{\frac{T}{2}} ds \int_{-\frac{T}{2}}^{\frac{T}{2}} dt\, \langle x(s)x(t)\rangle\, e^{-j\omega(s-t)}$$

$$= \int_{-\frac{T}{2}}^{\frac{T}{2}} ds \int_{-\frac{T}{2}}^{\frac{T}{2}} dt\, C(s - t) e^{-j\omega(s-t)}$$

Since the integrand depends only on the variable s-t one can do one of the standard manipulations in multivariable calculus that occurs often. Please show that for any integrable function g

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} ds \int_{-\frac{T}{2}}^{\frac{T}{2}} dt\, g(s - t) = \int_{-T}^{T} d\tau g(\tau)\, (T - |\tau|)$$

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Where we have defined τ=s-t.

Thus we obtain (identifyIng $g(\tau) = e^{-jw\tau}C(\tau)$)

$$\langle|\hat{x}_T(\omega)|\rangle^2 = \int_{-\mathrm{T}}^{T} d\tau e^{-j\omega t} C(\tau)\,(T - |\tau|)$$

At this point one can be an optimistic physists, divide by T and let T→ ∞ and obtain the required result.

III. RESULTS AND DISCUSSION

The following are the MATLAB simulated outputs to prove weiner-kinchin relation.

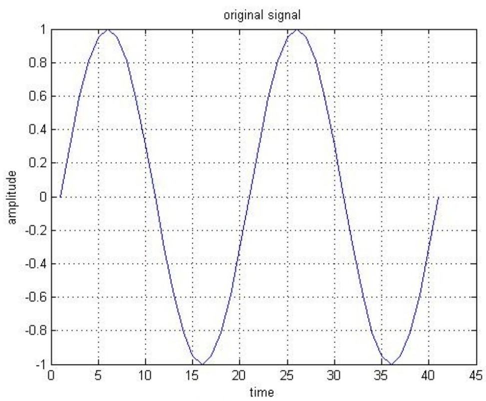

Fig.1. sine signal

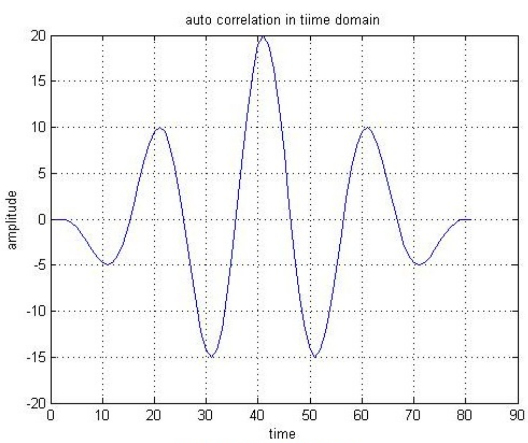

Fig.2. Auto correlatio of sine

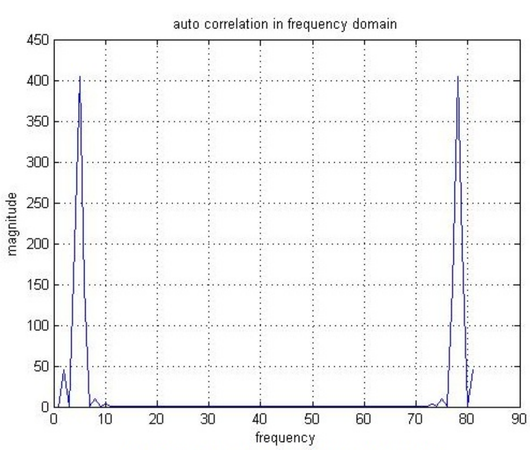

Fig.3. Fourier Transform of auto correlation of sine

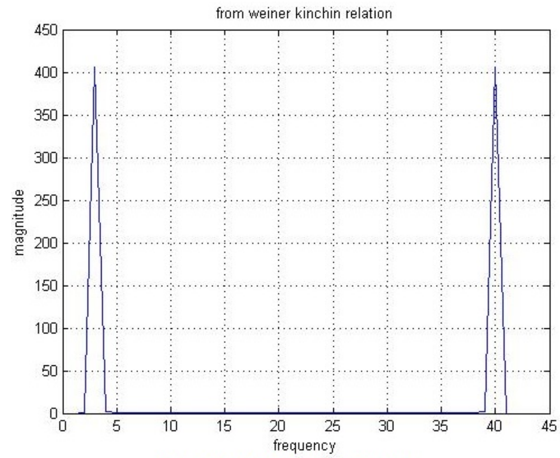

Fig.4. PSD calculated from FT(sine)^2

IV.CONCLUSION

We have proved the Wiener-Khinchin result for continuous time signal. This has been combined with Gardner's result to prove that Nyquist's criterion can be used to subsample a certain class of PSD-bandlimited nonstationary signals. Application of these two results to subsampling a simulated time sequence of spatially non-WSS signals is shown.

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