PERFORMANCE EVALUATION OF NESTED COSTAS CODES FOR IMPROVING RANGE RESOLUTION

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Abstract
In this paper, a modified Costas code is proposed based on the orthogonality condition. If the time band width product is increased beyond the ortogonality in Costas code, the unwanted spikes known as grating lobes are appeared. In this work, Costas code is nested with phase codes using kronecker product. The grating lobes are nullified and sidelobes also decreased. The performance evaluation of Nested Costas codes are carried out for different time band width products to obtain good range resolution.
Key words- Costas code, Phase codes, Autocorrelation Function, Grating lobes, Sidelobes and Range resolution.

I. INTRODUCTION

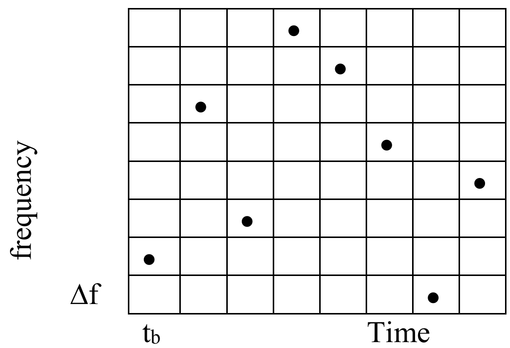
In multiple target environments high range resolution is required to detect target exact location. Indeed pulse compression is used in radar [1][2]. In pulse compression frequency or phase modulation is used to increase the band width before the transmitting the signal and reference signal is send to receiver. Received signal is autocorrelated with the reference signal to get the information about the target [3]. In phase and frequency modulations different signals are used, such as Barker codes, Poly phase codes, m-sequence codes, Golomb codes, Px code, Linear Frequency Modulation (LFM), Non-Linear Frequency Modulation (NLFM), Stepped Frequency Train of LFM(SLFM) and Costas codes...etc[4-7].

The Costas codes are capable of improving the Ambiguity function in range and Doppler directions [4]. In [5, 7, 8], the authors proposed different algorithms to construct Costas codes. From all the approaches the sequence of length M approaches Costas properties by using the difference matrix method [9]. When time band width product of Costas code is equal to one, there are no grating lobes in range axis [10-14]. If time bandwidth product $t_b \Delta f > 1$, the unwanted peaks (grating lobes) are appeared in delay axis and decrease Peak Side Lobe Ratio (PSLR). To nullify the grating lobes, author in [15], modulate the Costas sub-pulses with LFM ones. Grating lobes are nullified by using search processing but side lobes are remaining. In [16], phase codes are introduced in Costas sub-pulses to overcome the grating lobes and side lobes problem. In the proposed method, phase codes such as Barker code and poly phase codes (Frank code P3 and P4 code) are nested with Costas code using Kronecker product to nullify the grating lobes and thereby improve the PSLR.

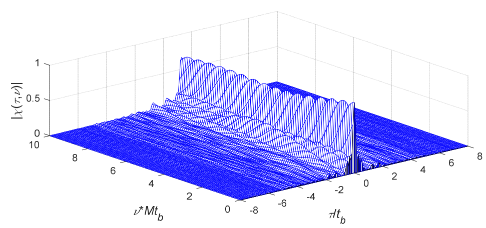
This paper is organized as, section II, Construction of Costas code and its Ambiguity Function (AF). Section III gives the information about phase codes. Section IV relates the grating lobes and sub-pulse coding. In section V, Costas code is nested with Phase codes using Kronecker product to nullifying the grating lobes. Section VI, gives the conclusions.

II. COSTAS CODE

Costas waveform [4] is generated by a long pulse width T into a series of M sub pulses, where the frequency of each sub-pulse is selected from M contiguous frequencies within a band. In Costas codes the frequencies for the sub-pulses are selected randomly. For this purpose, consider the M×M matrix shown in Fig. 1(a), the size of Costas code is M= 8(2, 6, 3, 8, 7, 5, 1, 4). It allows better approximation of ideal Ambiguity function.



(a)



(b)

Fig. 1. (a) Costas code M=8(2, 6, 3, 8, 7, 5, 1, 4). (b) Ambiguity Function of Costas code M=8

The Costas signal corresponding to the code $[a_1,a_2\ldots\ldots a_m]$ is given by:

$$x(t) = \sum_{m=1}^{M} u_m(t-(m-1)t_b). \qquad (1)$$

Where $u_m = exp(j2\pi f_m t)\ X\ Rect\left(\frac{t}{t_b}\right)$

$$Rect\left(\frac{t}{t_b}\right) = \begin{cases} 1 \text{ if } 0 \le t \le t_b \\ 0 \quad \text{elsewhere} \end{cases}$$

For simple Costas signal

$$f_m = a_m \Delta f = a_m/t_b$$

A. Performance measure

The performance measure of Pulse Compression is Peak side lobe ratio (PSLR). It is the ratio of the maximum of the sidelobe level to maximum mainlobe level.

$$\text{PSLR (dB)} = 20 \log_{10} \frac{max_{1\le k\le N}|R(k)|}{|R(0)|}. \qquad (2)$$

Where R(0) is the mainlobe level and R(k) is maximum side lobe level among all sidelobes. The Ambiguity function for Costas code M=8 is shown in figure. 1 (b).

III. PHASE CODES

B. Barker code

The highest length of Barker code is 13. The length of the Barker code is the ratio of the mainlobe to the sidelobe level in the autocorrelation function and the side lobes are '1' or '-1' [17].

Barker code 13 = [1,1,1,1,1,-1,-1,1,1,-1,1,-1,1]

C. P3 and P4 Code

P4 [18-20] is having the smallest phase increments from sample to sample on the center of the waveform. The P4 code is more Doppler tolerant and better precompression bandwidth limiting than the P3 code. It is constructed with any length of N. P3 code 16 is given as

□□□□ $\left[0\ \frac{\pi}{16}\ \frac{4\pi}{16}\ \frac{9\pi}{16}\ \pi\ \frac{25\pi}{16}\ \frac{4\pi}{16}\ \frac{17\pi}{16}\ 0\ \frac{17\pi}{16}\ \frac{4\pi}{16}\ \frac{25\pi}{16}\ \pi\ \frac{9\pi}{16}\ \frac{4\pi}{16}\ \frac{\pi}{16}\right]$

P4 code 16 is given as

□□□□ $\left[0\ \frac{17\pi}{16}\ \frac{4\pi}{16}\ \frac{25\pi}{16}\ \pi\ \frac{9\pi}{16}\ \frac{4\pi}{16}\ \frac{\pi}{16}\ 0\ \frac{\pi}{16}\ \frac{4\pi}{16}\ \frac{9\pi}{16}\ \pi\ \frac{25\pi}{16}\ \frac{4\pi}{16}\ \frac{25\pi}{16}\right]$

D. Frank Code

A Frank code of N^2 sub-pulses are referred to as an N-phase Frank code [21]. The first step in computing a Frank code is to divide 360^0 by N, and define the result as the fundamental phase increment Δφ.

$$\Delta\varphi = \frac{360^0}{N} \qquad (3)$$

For N-phase Frank code the phase of each sub-pulse is computed from

$$\begin{pmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 2 & 3 & \ldots & N-1 \\ 0 & 2 & 4 & 6 & \ldots & 2(N-1) \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & (N-1) & 2(N-1) & 3(N-1) & \cdots & (N-1)^2 \end{pmatrix} \Delta\phi$$

(4)

Where each row represents a group and a column represents the sub-pulses for that group. For example, a 4-phase Frank code has N=4, and the fundamental phase increment is $\Delta\varphi = \frac{360^0}{4} = 90^0$. It follows that

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 90^0 & 180^0 & 270^0 \\ 0 & 180^0 & 0^0 & 180^0 \\ 0 & 270^0 & 180^0 & 90^0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{pmatrix}$$
(5)

Therefore, a Frank code of 16 elements is given by

F_{16}={1 1 1 1 1 j –1 –j 1 –1 1 –1 1 –j –1 j}

The phase of the i^{th} code element in the j^{th} row or code group may be expressed mathematically as

$$\Phi_{Frank}^{i,j}=(2\pi/N)(i-1)(j-1) \quad (6)$$

Where i = 1, 2, 3,..N and j = 1, 2, 3, ..., N. In above equation the index i ranges from 1 to N for each value of j and the number of code elements formed is equal to N^2. The Ambiguity plot of Frank Code of length 16 is shown in figure 2.

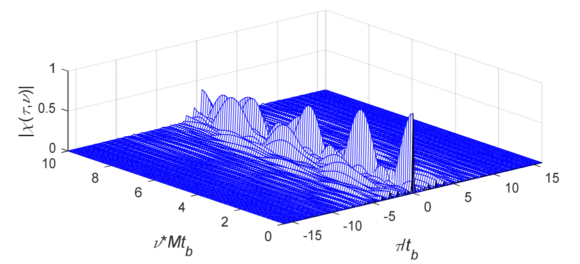


Fig. 2. Ambiguity plot for Frank Code of length 16.

IV. COSTAS SUB-PULSE CODING AND GRATING LOBES

Costas with LFM (Linear Frequency Modulation) is having low PSLR and large main lobe width when it obeys the orthogonality condition. In Costas codes, when $t_b\Delta f > 1$ will give grating lobes corresponding to the value of $t_b\Delta f$ shown in figure 3, where X axis represents the delay and Y axis represents the normalized Autocorrelation. The Autocorrelation function $R(\tau)$ is defined by the product of two functions, $R_1(\tau)$ and $R_2(\tau)$ [15]. $R_1(\tau)$ indicates autocorrelation function of the code and $R_2(\tau)$ indicates grating lobes distribution shown in equation number 7.

$$R\left(\frac{\tau}{t_b}\right) = \left|R_1\left(\frac{\tau}{t_b}\right)\right|\left|R_2\left(\frac{\tau}{t_b}\right)\right|$$

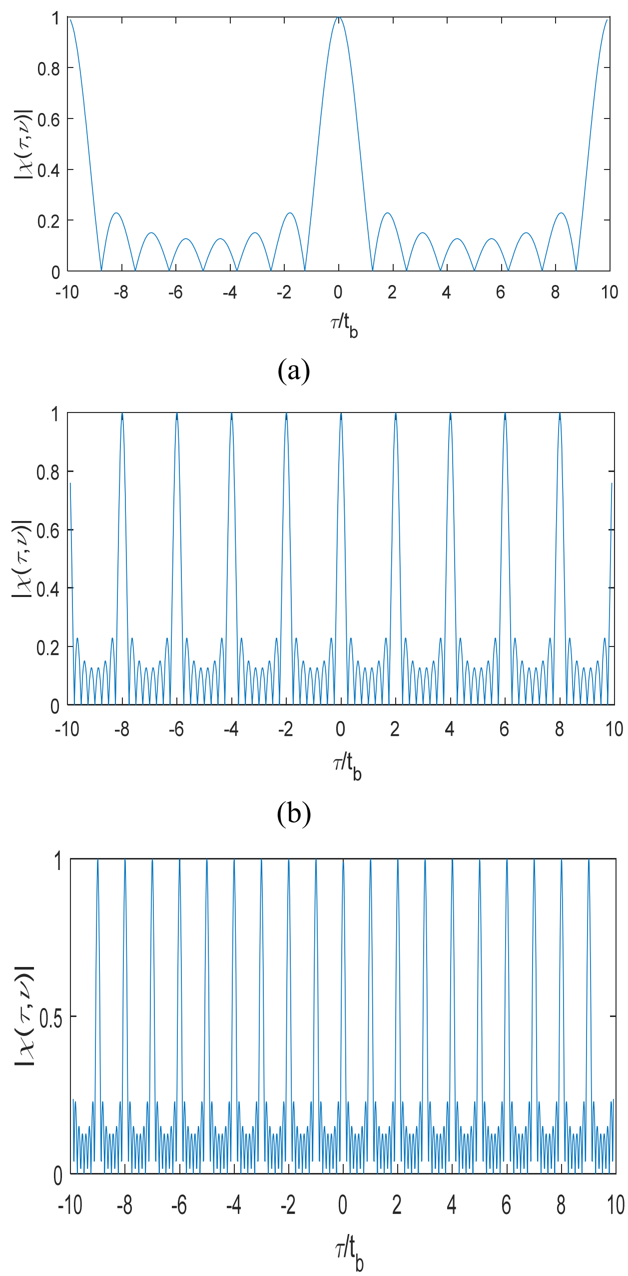
$$= \left|R_1\left(\frac{\tau}{t_b}\right)\right| \cdot \left|\frac{\sin\left(M\pi t_b\Delta f\frac{\tau}{t_b}\right)}{M\sin\left(\pi t_b\Delta f\frac{\tau}{t_b}\right)}\right|; \qquad \tau<t_b \quad (7)$$

The grating lobes are appearing at maxima of $R_2(\tau)$.

$$\tau = \frac{k}{t_b\Delta f}t_b$$

$$k=0, \pm1, \pm2, \dots\ |t_b\Delta f| \quad (8)$$

In [16], the author proposes the phase codes with good correlation properties in the sub-pulses of Costas code, to lower the level of grating lobes and sidelobes. The PSLR values for sub-pulse coding is tabulated in table 1.



(a)

(b)

(c)

Fig. 3. Autocorrelation Function plots for (a).

$t_b\Delta f$=1, (b). $t_b\Delta f$=5, (c) $t_b\Delta f$=10

E. Kronecker Product

Nested codes can be obtained by using the Kronecker product of two codes [2,22] whose initial matched filter response is good. The Kronecker product is denoted by ⊗, is an operation on two matrices of arbitrary size resulting in a block matrix. 'C' is an r x s matrix and 'D' is a v × u matrix, then the Kronecker product C ⊗D is the rv × su block matrix.

$$C \otimes D = \begin{bmatrix} C_{11}D & \cdots & C_{1s}D \\ \vdots & \ddots & \vdots \\ C_{r1}D & \cdots & C_{rs}D \end{bmatrix} \quad (9)$$

The Costas code of length 8 is considered as 'C' and another one is Frank 16 is considered as 'D'.

$C \otimes D$ = [2,6,3,8,7,5,1,4]⊗[1,1,1,1,1,j,-1,-j,1,-1,1,-1,1,-j,-1,j]

In the proposed approach, Costas code is nested with Frank code 16 using Kronecker product. The grating lobes are suppressed and peak sidelobes also decreased shown in figures 4-6. When peak side lobes are decreased false alarm is avoided. The range resolution is also increased because of increasing the phase values in Costas code by Frank 16. The PSLR values for all cases are tabulated for Nested Costas with Frank code 16 in table 1. When $t_b\Delta f$=1 the PSLR is obtained for Nesting of Costas with Frank code 16 is -39.3 dB. For $t_b\Delta f = 5$ and $t_b\Delta f = 10$, the correspond PSLR are -41.0 dB and -44.6 dB.

The same approach is applied for Nesting of Costas with Barker code 13, P3 code 16 and P4 code 16 for $t_b\Delta f = 1$, $t_b\Delta f = 5$ and $t_b\Delta f = 10$. The figures 7-9 represents the Autocorrelation Function plots for nesting of Costas codes with Barker 13 code. In this case maximum PSLR -27.7 dB is obtained for $t_b\Delta f = 5$.

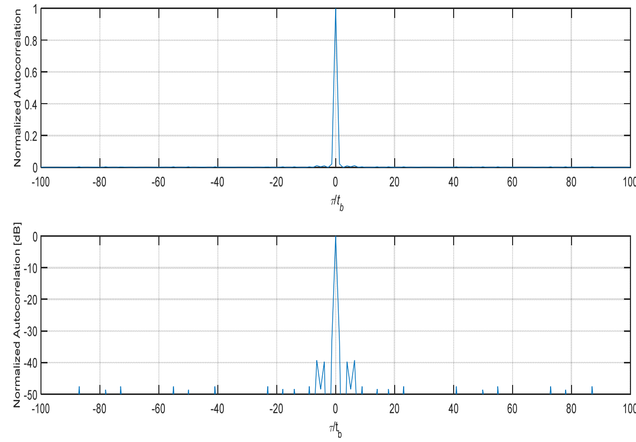


Fig. 4. Autocorrelation Function plot for Costas-Frank 16 for $t_b\Delta f$=1. Top: Normalized ACF. Bottom: Normalized ACF in dB.

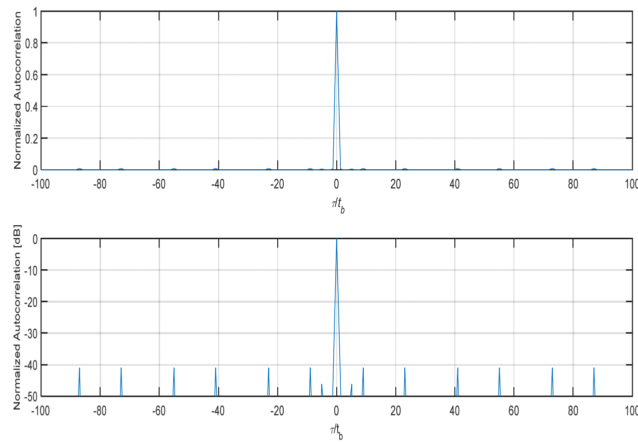


Fig. 5. Autocorrelation Function plot for Costas-Frank 16 for $t_b\Delta f$=5. Top: Normalized ACF. Bottom: Normalized ACF in dB.

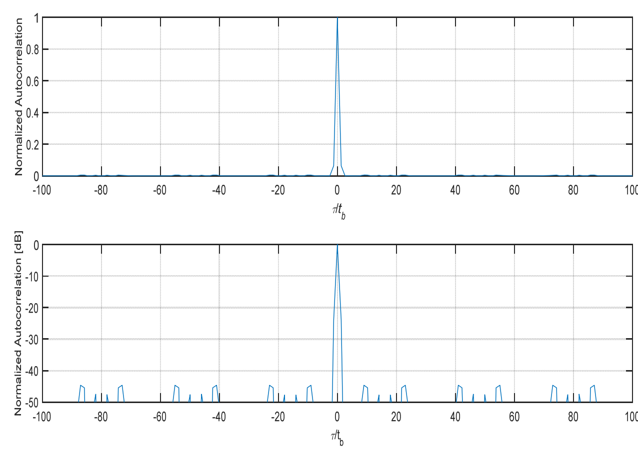


Fig. 6. Autocorrelation Function plot for Costas-Frank 16 for $t_b\Delta f$=10. Top: Normalized ACF. Bottom: Normalized ACF in dB.

Figures 10-12 represents the Autocorrelation Function plots for nesting of Costas code with P3 length 16 code. The corresponding PSLR values are tabulated in table 1 and compared with Costa sub-pulse coding. The Highest PSLR is obtained at $t_b\Delta f$=10 is -39.7 dB when compared to the time bandwidth products $t_b\Delta f$=1 and $t_b\Delta f$=5.

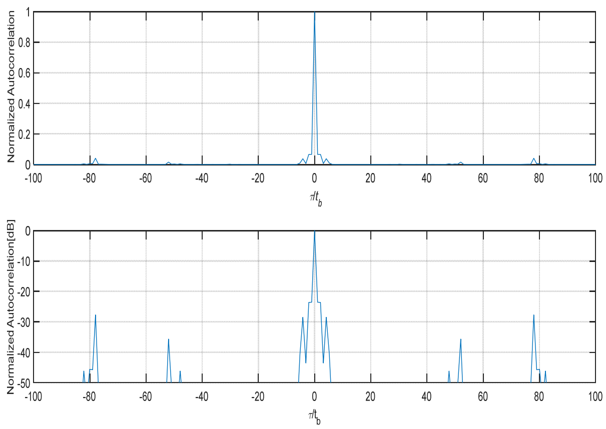


Fig. 7. Autocorrelation Function plot for Costas-Barker 13 for $t_b\Delta f$=1. Top: Normalized ACF. Bottom: Normalized ACF in dB.

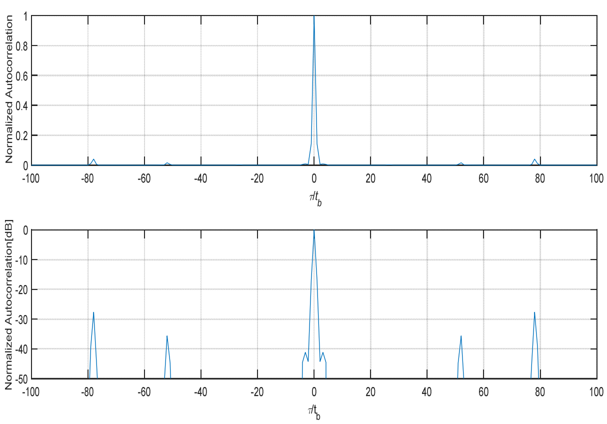


Fig. 8. Autocorrelation Function plot for Costas-Barker 13 for $t_b\Delta f$=5. Top: Normalized ACF. Bottom: Normalized ACF in dB.

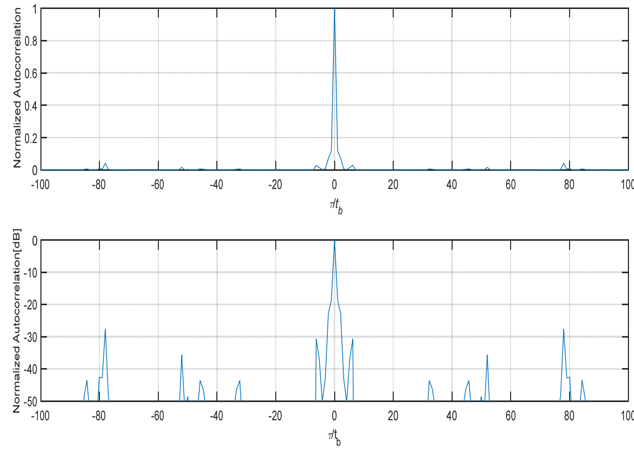


Fig. 9. Autocorrelation Function plot for Costas-Barker 13 for $t_b\Delta f$=10. Top: Normalized ACF. Bottom: Normalized ACF in dB.

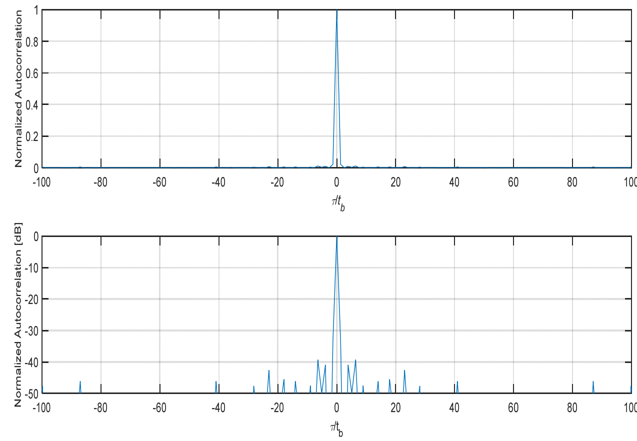


Fig. 10. Autocorrelation Function plot for Costas-P3 16 for $t_b\Delta f$=1. Top: Normalized ACF. Bottom: Normalized ACF in dB.

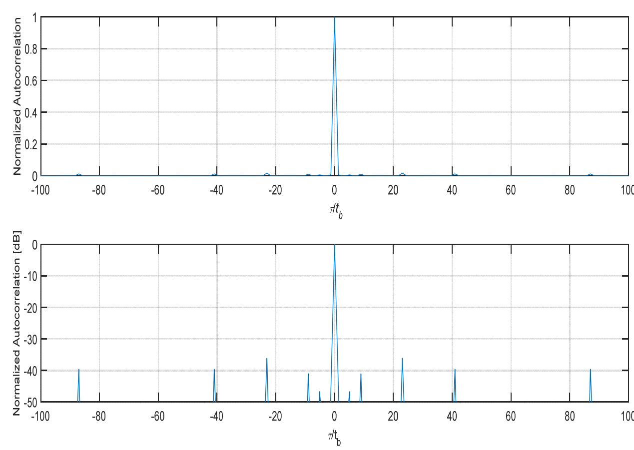


Fig. 11. Autocorrelation Function plot for Costas-P3 16 for $t_b\Delta f$=5. Top: Normalized ACF. Bottom: Normalized ACF in dB.

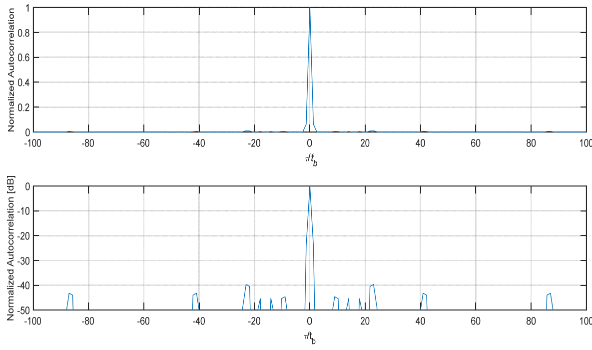


Fig. 12. Autocorrelation Function plot for Costas-P3 16 for $t_b\Delta f$=10. Top: Normalized ACF. Bottom: Normalized ACF in dB.

Figures 13-15 represents the Autocorrelation Function plots for nesting of Costas code with P4 length 16 code. The corresponding PSLR values are tabulated in table 1 and compared with Costa sub-pulse coding. The maximum PSLR is -42.6 dB is achieved for $t_b\Delta f$=1, when compared to the time bandwidth products $t_b\Delta f$=1 and $t_b\Delta f$=5.

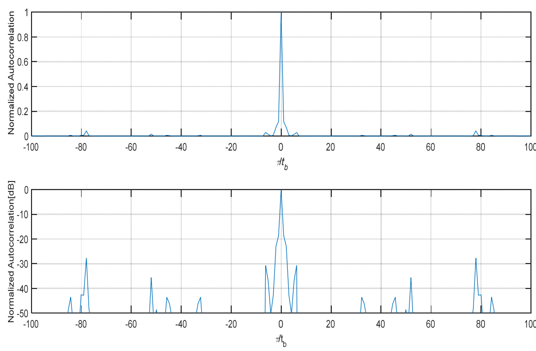


Fig. 13. Autocorrelation Function plot for Costas-P4 16 for $t_b\Delta f$=1. Top: Normalized ACF. Bottom: Normalized ACF in dB.

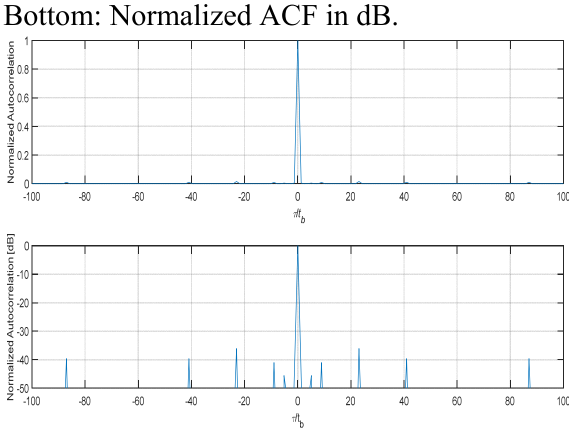


Fig. 14. Autocorrelation Function plot for Costas-P4 16 for $t_b\Delta f$=5. Top: Normalized ACF. Bottom: Normalized ACF in dB.

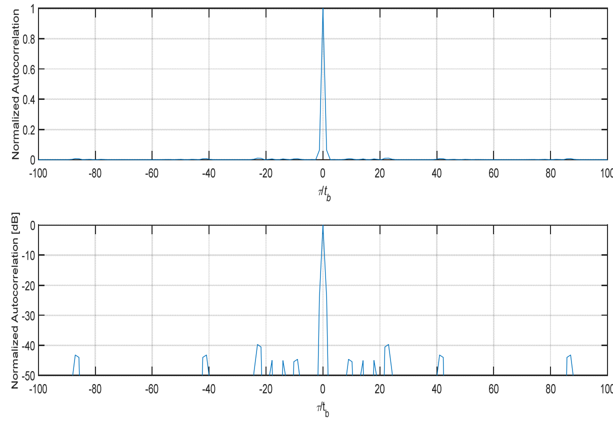


Fig. 15. Autocorrelation Function plot for Costas-P4 16 for $t_b\Delta f$=10. Top: Normalized ACF. Bottom: Normalized ACF in dB.

Table. 1. Comparison of PSLR for Costas sub pulse coding and nested Costas coding

Phase Codes	Costas Sub- pulse Coding PSLR			Nested Costas Coding PSLR		
	$t_b.\Delta f = 1$	$t_b.\Delta f = 5$	$t_b.\Delta f = 10$	$t_b.\Delta f = 1$	$t_b.\Delta f = 5$	$t_b.\Delta f = 10$
Frank 16	-17.2 dB	-19.2 dB	-23.3 dB	-39.3 dB	-41.0 dB	-44.6 dB
Barker 13	-14.1 dB	-19.8 dB	-23.6 dB	-23.6 dB	-27.7 dB	-22.9 dB
P_3 16	--	--	--	-39.3 dB	-36.0 dB	-39.7 dB
P_4 16	-16.7 dB	-18.7 dB	-20.9 dB	-42.6 dB	-36.1 dB	-39.7 dB

VI. CONCLUSIONS

In Costas code, the time band width product is increasing beyond orthogonality condition to get high range resolution in multiple target environment. When time band width is increases, grating lobes are presented. In this work, Frank code 16 , Barker code 13, P3 code 16 and P4 code 16 are nested with Costas code using Kronecker product to mitigate the grating lobes and decreasing the sidelobes. When Costas code is nested with Frank code of length 16, the PSLR = -44.6 dB is obtained for $t_b\Delta f$=10. For Costas-Barker code of length 13, the PSLR = -27.7 dB is obtained for $t_b\Delta f$=5. With Costas-P3 16, the PSLR = -39.7 dB is obtained for $t_b\Delta f$=10. The Costas-P4 16, the PSLR = -42.6 dB is obtained for $t_b\Delta f$=1. The performance of nesting of Costas code with Frank code is shown better PSLR when compared to P3, P4 and Barker codes. When compared to Costas sub-pulse coding improved PSLR is obtained for Nested Costas coding.

REFERENCES.

[1] Merrill I. Skolnik, "Introduction to Radar System.(3rd ed.)," NewYork: McGraw-Hill, 2002.

[2] N. Levanon and E. Mozeson, Radar signals. John Wiley and Sons, Inc, 2004.

[3] Skolnik, M.: 'Radar Hand book'(McGraw-Hill Education, 2008, 3rd end.).

[4] J. Costas, "A study of a class of detection waveforms having nearly ideal range-doppler ambiguity properties," Proceedings of the IEEE, vol. 72, no. 8, , August 1984, pp. 996–1009.

[5] Golomb, S. W., and H. Talar, "Constructions and properties of Costas Arrays'' Proceedings of the IEEE, vol. 72, no. 9, September 1984, pp. 1143–1163.

[6] T.D. Bhatt, E. G. Rajan, P. V. D. Somasekhar Roa 'Design of frequency coded waveforms for target detection'' IET Radar Sonar Navig., 2008, Vol. 2, NO. 5, pp. 388-394.

[7] James K. Bread, Jon C. Russo, Keith G. Erickson, Michael C. Monteleone, Michael T.Wright, "Costas Array Generation and Search Methodology" IEEE Transaction on Aerospace and Electronics systems, 43, NO. 2 (Apr.2007),522-538.

[8] Golomb, S.: The status of array construction'. 40th annual Conf. On Information Sciences and Systems, March 2006, pp. 522-524.

[9] Drakakis, K.: ' A review of Costas arrays,' J. Appl. Math., 2006, pp. 1-32.

[10] Freedman, A., Levanon, N.: 'Staggered Costas signals.' IEEE Trans. Aerosp. Electron. Syst., 1986, AES-22, (6), pp. 695-702.

[11] Levanon, N., Mozeson, E.: 'Orthogonal train of Modified Costas pulses.' Proc. of the IEEE Radar Conf., April 2004, pp. 255-259.

[12] Y. Hongbing, Z. Jianjiang, W. Fei, and Z. Zhenkai, "Design and analysis of Costas/PSKRF stealth signal waveform," in Radar (Radar), 2011 IEEE CIE International Conference on, vol. 2, Oct., pp. 1247–1250.

[13] P. Pace and C. Y. Ng, "Costas CW frequency hopping radar waveform: peak sidelobe improvement using Golay complementary sequences," Electronics Letters 2010, vol. 46, no. 2, pp. 169–170.

[14] J. Lemieux and F. Ingels, "Analysis of FSK/PSK modulated radar signals using Costas arrays and complementary Welti codes," Record of the IEEE int. Radar Conf., 1990, May, pp. 589–594.

[15] N. Levanon and E. Mozeson, "Modified Costas signal," Aerospace and Electronic Systems, IEEE Transactions on, vol. 40, no. 3, pp. 946–953, July.2004.

[16] Nadjah TOUATI, Charles TATKEU, Thierry CHONAVEL and Atika RIVEN, "Phase Coded Costas Signals for Ambiguity Function Improvement and Grating lobes Suppression", IEEE 78th Vehicular Tech. Conf. (VTC Fall), 2013, pp. 1-5.

[17] Kerdock. A.M. R Mayer and D. Bass. "Longest Binary Pulse Compression Codes with given peak sidelobe levels." Proceeding of IEEE. Vol. 74.,No.2,p-366, Feb 1986.

[18] Frank, R. L. " Polyphase codes with good non periodic correlation properties". IEEE Transactions on information theory, IT-9 (Jan. 1963), 43-45.

[19] Lewis, B. L. and F. F. Kretschmer, Jr., " A New class of Polyphase pulse compression codes and techniques," IEEE Transactions on Aerospace and electronic system, Vol. 17, No. 3, 364-372, May 1981.

[20] Lewis, B. L. and F. F. Kretschmer, Jr., " Linear frequency modulation derived Polyphase pulse compression codes," IEEE Transactions on Aerospace and electronic system, Vol. 17, 637-641, Sep. 1982.

[21] L. Lewis and F. Kretschmer, "A new class of polyphase pulse compression codes and techniques," Aerospace and Electronic Systems,IEEE Transactions on, vol. AES-17, no. 3, pp. 364–372, 1981.

[22] N. Levanon "cross-correlation of long binary signals with longer mismatched filter" IEEE Proc. Radar Sonar Navigation, Vol 152, no. 6.pp 377-382, December 2005.