A STUDY ON STRUCTURES OF MODIFIED FUZZY AND ANTI FUZZY GRAPHS

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Abstract
Graphs are simple model of relation. It is a convenient way of representing information involving relationship between objects. The object is represented by vertices and relations by edges. When there is vagueness in the description of the objects or in its relationships or in both we need to design fuzzy graph model which is of recent interest. In this paper, some results related to modified fuzzy graphs and modified anti fuzzy graphs are introduced. Suitable examples are illustrated to demonstrate the proposed notion.
Keywords: Modified fuzzy graph, Modified anti fuzzy graph, Complement of modified fuzzy and anti fuzzy graph, Size, Order, degree of modified fuzzy graph, Regular and totally regular modified fuzzy graph, Irregular and totally irregular modified fuzzy graph, Very totally regular and Very totally irregular modified fuzzy graph, Open neighborhood degree, Closed neighborhood degree, mem-measures.
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1. Introduction

In 1736, Euler first introduced the notion of graph theory. In the history of mathematics, the solution given by Euler for the well known Konigsberg bridge problem is considered to be the first theorem of graph theory. It is noted that many real world systems can be modeled using graphs. Any entity involving points and connections between them is called as graph. The connections may be physical as in electrical networks and computer networks or relationships as in molecules and ecosystems. Graph theory has several interesting applications in system analysis, operations research, economics etc. Since most of the time, the aspects of graph problems are uncertain, it is nice to deal with these aspects via the methods of fuzzy set theory.

Fuzzy set theory was introduced by Zadeh [9] in his Landmark paper "Fuzzy sets" in 1965.

Fuzzy graph and several fuzzy analogs of graph theoretic concepts were introduced by Azirel Rosenfeld [7] in 1975, whose basic idea was introduced by Kauffmann [5] in 1973. Though it is very young, it has been growing fast and has numerous applications in various fields. Moderson [6] and Sunitha [8] defined the concept of complement of fuzzy graphs and studied some operations on it. Nagoor Gani [3,4] introduced regular fuzzy graphs and irregular fuzzy graphs. Akram[1] introduced anti fuzzy structures on graphs. Elizabeth and Sujatha[2] introduced a definition for modified fuzzy graph and modified anti fuzzy graphs. Thus numerous papers have been published under fuzzy graphs.

The paper is organized as follows: In section 2, we review the basic definitions related to graph, modified fuzzy graph and modified anti fuzzy graph. Section 3, focus on some important results related to modified fuzzy structures and modified anti fuzzy structures on graphs. The paper is concluded in section 4.

2. Pre- requisites

Definition 2.1. Crisp Graph G*

G*(V,E), where V is the set and E is a relation on V. The elements of V are vertices of G* and the elements of E are edges of G*.

Definition 2.2. Neighborhood of a vertex in G*

The neighborhood of a vertex v in a graph G* consist of all vertices adjacent to v and all edges connecting two such vertices. The neighborhood is often denoted by N (v). The degree deg (v) of vertex v is the number of edges incident on v. The set of neighbors, called a open neighborhood N(v) for a vertex v in a graph G*,consists of all vertices adjacent to v but not including v, that is, $N(v)=\{u \in V/vu \in E\}$. When v is also included, it is called a closed neighborhood N[v], that is, $N[v] = N(v) \cup \{v\}$.

Definition 2.3. Connected Graph G*

An undirected graph G* is connected if there is a path between each pair of distinct vertices.

Definition 2.4. Path, length, cycle in G*

A path in a graph G^* is a sequence of vertices such that from each of its vertices there is an edge to the next vertex in the sequence. The length of a path P: $v_1,v_2,\ldots v_{n+1}$ (n>0) in G^* is n. A path

P: $v_1v_2,\ldots v_{n+1}$ in G is called a cycle if $v_1 = v_{n+1}$ and $n\geq 3$.

Definition 2.5. Distance between a vertices in a connected graph G*

For a pair of vertices u, v in a connected graph G^*, the distance d(u, v) between u and v is the length of a shortest path connecting u and v.

Definition 2.6. Eccentricity of a vertex in a connected graph G*

The eccentricity e(v) of a vertex v in a graph G^* is the distance from v to a vertex furthest from v, that is, $e(v) = \max\{d(u, v) \mid u \in V\}$.

Definition 2.7. Radius of a connected graph G*

The radius of a connected graph (or weighted graph) G^* is defined as $rad(G^*) = \min\{e(v) \mid v \in V\}$.

Definition 2.8. Diameter of a connected graph G*

The diameter of a connected graph (or weighted graph) G^* is defined as $diam(G^*) = \max\{e(v) \mid v \in V\}$. The eccentric set S of a graph is its set of eccentricities.

Definition 2.9. Self centered graph

The center $C(G^*)$ of a graph G^* is the set of vertices with minimum eccentricity. A self-centered graph is a graph whose diameter equals its radius.

Definition 2.10. Fuzzy relation and Fuzzy graph

A fuzzy subset σ on a set X is a map $\sigma : X \to [0,1]$. A fuzzy binary relation on X is a fuzzy subset σ on X*X. By a fuzzy relation we mean a fuzzy binary relation given by $\sigma : X*X \to [0,1]$. Fuzzy graph is a graph consists of a pairs of vertex and edge that have degree of membership containing closed interval of real number [0,1] on each edge and vertex.

Definition 2.11. Modified fuzzy graph [2**]**

Let V be the set of vertices and E be the set of edges. A modified fuzzy graph G of a crisp graph $G^*(V,E)$ we mean a pair $G = (\sigma, \mu)$, where σ is a fuzzy set on V and μ is a fuzzy relation on E such that $\mu(x,y) \leq \sigma(x) \vee \sigma(y)$, for all $x,y \in V$ and $xy \in E$.

Definition 2.12. Modified Anti fuzzy graph [2]

An anti fuzzy graph G of a graph $G^*(V,E)$, we mean a pair $G = (\sigma, \mu)$, where σ is a fuzzy set on V and μ is an anti fuzzy relation on E such that $\mu(x,y) \geq \sigma(x) \wedge \sigma(y)$, for all $x, y \in V$ and $xy \in E$.

3. Modified fuzzy and Anti fuzzy structures on graphs

Notation: Throughout this paper we denote the modified fuzzy graph (MFG) as G: (σ_G, μ_G) and modified anti fuzzy graph (MAFG) as G : $(\sigma_{G'}, \mu_{G'})$

Definition 3.1. Complete modified fuzzy graph

A MFG G is said to be complete if $\mu_G(x,y) = \sigma_G(x) \vee \sigma_G(y)$, for all $x,y \in V$.

Definition 3.2. Complete modified anti fuzzy graph

A MAFG G' is said to be complete if $\mu_{G'}(x,y) = \sigma_{G'}(x) \wedge \sigma_{G'}(y)$, for all $x,y \in V$.

Theorem 3.1. Let G: (σ_G, μ_G) be a MFG and let G^c: $(\sigma_G{}^c, \mu_G{}^c)$ be a complement of a MFG where $\sigma_G{}^c \equiv \sigma_G$ then $\mu_G{}^c = 0$ if $0 < \mu_G(x,y) \leq 1$

$\mu_G{}^c = \sigma_G(x) \vee \sigma_G(y)$, if $\mu_G(x,y) = 0$

Proof. Consider $\mu_G(x,y) = \sigma_G(x) \vee \sigma_G(y)$

$$\mu_G{}^c(x,y) = \sigma_G{}^c(x) \vee \sigma_G{}^c(y)$$
$$\mu_G{}^c(x,y) = \sigma_G(x) \vee \sigma_G(y)$$

Definition 3.3. Alternative complement of MFG.

Let G: (σ_G, μ_G) be a MFG and let $\bar{G}$: $(\bar{\sigma}_G,\bar{\mu}_G)$ where $\bar{\sigma}_G \equiv \sigma_G$ and $\bar{\mu}_G(x,y)= \sigma_G(x) \vee \sigma_G(y)-\mu_G(x,y)$ for $0 \leq \mu G(x,y) \leq 1$.

Theorem 3.2. $\bar{\bar{\mu}}_G(x,y) = \mu_G(x,y)$ for all x,y in MFG

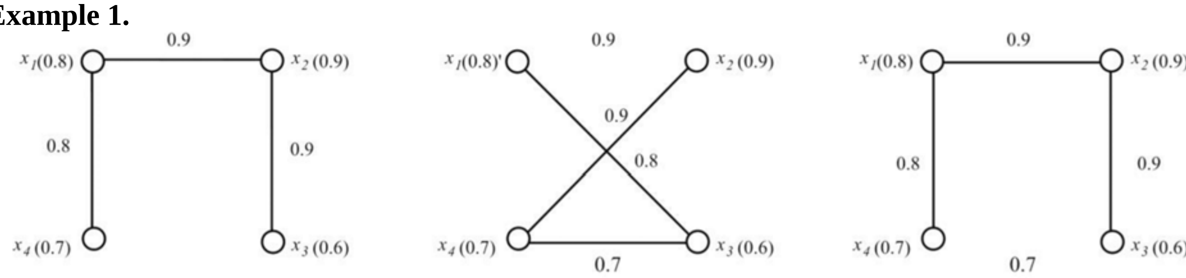
Proof. $\bar{\mu}_G(x,y) = \sigma_G(x) \vee \sigma_G(y)-\mu_G(x,y)$

$$\bar{\bar{\mu}}_G(x,y) = \bar{\sigma}_G(x) \vee \bar{\sigma}_G(y)-\bar{\mu}_G(x,y)$$
$$= \sigma_G(x) \vee \sigma_G(y)-(\sigma_G(x) \vee \sigma_G(y)-\mu_G(x,y))$$
$$= \sigma_G(x) \vee \sigma_G(y)-\sigma_G(x) \vee \sigma_G(y)+\mu_G(x,y)$$
$$= \mu_G(x,y)$$

Remark 1.

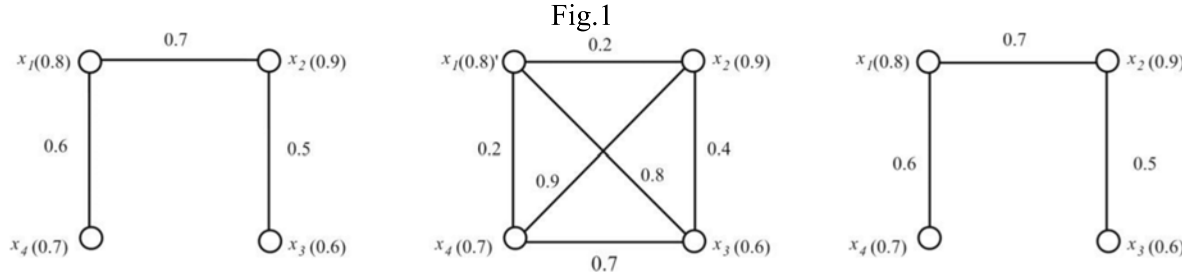
i) $(G^c)^c = G$, iff $\mu_G(x,y) = \sigma_G(x) \vee \sigma_G(y)$ in MFG. ii) $\bar{\bar{G}} = G$ iff $\mu_G(x,y) \leq \sigma_G(x) \vee \sigma_G(y)$ in MFG.

Example 1.



G G' (G')'

Fig.1

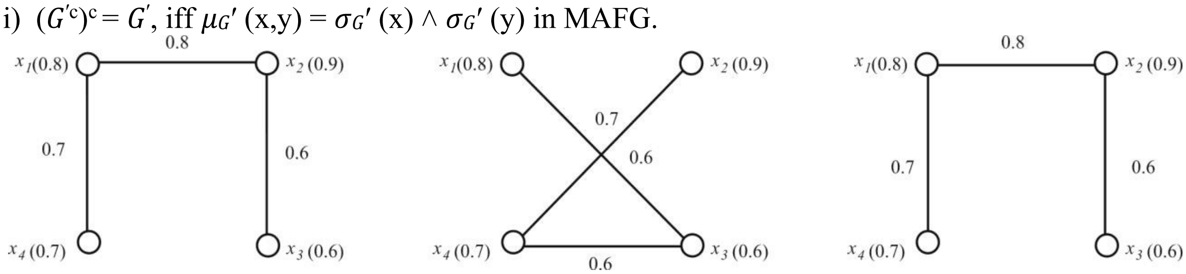


G $\bar{G}$ $\bar{\bar{G}}$

Fig.2

Remark 2.

i) $(G'^c)^c = G'$, iff $\mu_{G'}(x,y) = \sigma_{G'}(x) \wedge \sigma_{G'}(y)$ in MAFG.



G' G'^c $(G'^c)^c$

Fig.3

Definition 3.4: Degree of a modified fuzzy graph

Let G: (σ_G, μ_G) be a MFG. The degree of a vertex u is $d_G(u) = \sum_{u\neq v} \mu_G(uv)$. Since $\mu_G(uv) > 0$ for uv $\in E$ and $\mu_G(uv) = 0$, for uv $\notin E$, this is equivalent to $d_G(u) = \sum_{uv\in E} \mu_G(uv)$

The minimal degree of G is $\delta(G) = \wedge \{d_G(v)/v \in V\}$

The maximal degree of G is $\Delta(G) = \vee \{d_G(v)/v \in V\}$

Definition 3.5. Order of a MFG

The order of a MFG G is $O(G) = \sum_{u\in V} \sigma_G(u)$

Definition 3.6. Size of a MFG

The size of a MFG G is $S(G) = \sum_{uv\in E} \mu_G(uv)$

Definition 3.7. Total degree of a vertex of a MFG

Let G: (σ_G, μ_G) be a MFG . The total degree of a vertex u$\in$V is defined by

$Td_G(u) = \sum_{\substack{u\neq v\\ uv\in E}} \mu_G(uv) + \sigma_G(u)$

Example 2.

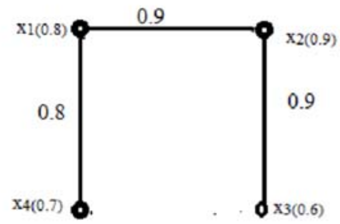


Fig.4: G

From fig.4, $d_G(x_1)$ = 0.8+0.9=1.7. Similarly, $d_G(x_2)$=1.8, $d_G(x_3)$=0.9 and $d_G(x_4)$=0.8.
$\delta(G) = 0.8$ and $\Delta(G) = 1.8$,
O(G)=3.0 and S(G)=2.6.
$td_G(x_1)$=1.7+0.8=2.5.
Similarly, $td_G(x_2)$=2.7 and $td_G(x_3)$=1.5 and $td_G(x_4)$=1.5.

Definition 3.8. k-regular MFG
Let G:(σ_G, μ_G) be a MFG, if $d_G(v)$=k, for all v∈ V(i.e) if each vertex has same degree k then G is said to be a k- regular MFG.

Definition 3.9. r-totally regular MFG
If each vertex of G has same total degree r, then G is said to be a r-totally regular MFG.
We state the following theorem without its proof which can be easily verified through example.

Theorem 3.3.
i) If μ_G is constant then MFG is k-regular. ii) If μ_G and σ_G are constant then MFG is k-regular and r-totally regular.

Example 3.

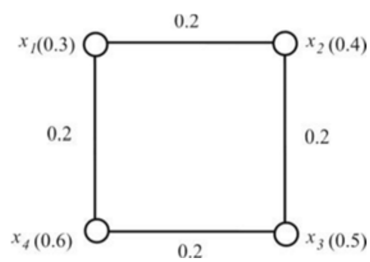


Fig.5: G

Here $d_G(v_i)$=0.4, for i=1,2,3,4. Therefore G is a 0.4-regular MFG.

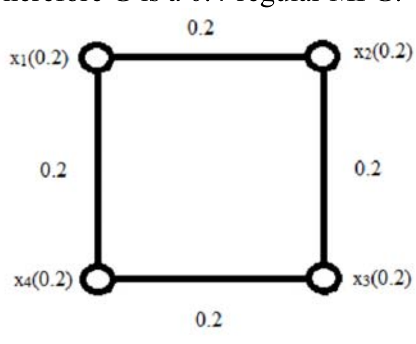


Fig.6: G Here

we find $d_G(v_i)$=0.4 and $td_G(v_i)$=0.6, for i=1,2,3,4

Therefore, G is 0.4-regular and 0.6-totally regular MFG.

Theorem 3.4. The sum of the order and size of a k-regular and r-totally regular MFG is
$O(G) + S(G) = p\left(\frac{2r-k}{2}\right)$ where p = |v|.

Proof. $O(G) + S(G) = p(\mu_G(x,y) + \sigma_G(x))$
$\mu_G(x,y) = \frac{k}{2}$, $\sigma_G(x) + 2\left(\frac{k}{2}\right) = r$,therefore $\sigma_G(x) = r - k$

O(G) + S(G) = $p\left(\frac{k}{2}+r-k\right)$

$= p\left(r-\frac{k}{2}\right) = p\left(\frac{2r-k}{2}\right)$

Example 4. For fig.6

O(G) + S(G) $= p\left(\frac{2r-k}{2}\right)$

$= 4\left(\frac{2(0.6)-0.4}{2}\right) = 4(0.4) = 1.6$

Definition 3.10. Regular MFG

Let G be a connected MFG. A regular MFG is a MFG where each vertex has the same number of open neighbors degree. That is, $D_G(x) = \sum_{y\in(x)} \sigma_G(y) = c$ (constant), where N(x) is the neighborhood of the vertex x. This happens only if σ_G is constant.

Note: The following example shows that there is no relationship between regular MFG and kregular MFG.

Example 5.

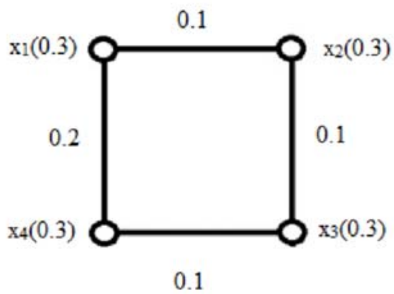


Fig.7:G

From fig.7, it is clear that G is regular MFG. Since each open neighbors degree is same, that is, 0.6. But it is not k-regular MFG since degree of each vertex is not same.

Definition 3.11. Irregular MFG

Let G be a connected MFG. If atleast one vertex of G has the distinct open neighbors degree, then G is an irregular MFG.

Definition 3.12. Highly irregular MFG

Let G be a connected MFG. G is called highly irregular MFG, if every vertex of G have distinct open neighbors degrees.

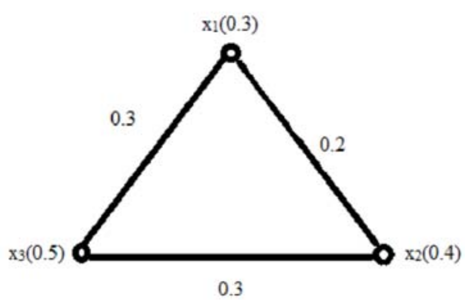


Fig.8: G

Definition 3.13. Totally regular MFG

Let G be a connected MFG. A totally regular MFG is a MFG, where each vertex has the same number of closed neighbors degree. That is, $D_G[x] = D_G(x) + \sigma_G(x)$ = constant.

Definition 3.14. Totally irregular MFG

Let G be a connected MFG. If atleast one vertex of G has distinct closed neighbors degree, then G is totally irregular MFG.

Definition 3.15. Highly totally irregular MFG

Let G be connected MFG. G is called highly totally irregular MFG, if every vertex of G have distinct closed neighbors degrees.

Definition 3.16. Very totally regular MFG

Let G be a connected MFG, where each vertex has the same number of very closed neighbors degree. That is, $D_G\{x\} = \sum_{y \in N(x)} \sigma_G(y) + \sigma_G(x) + \sum_{y \in N(x)} \mu_G(xy)$ = constant.

Definition 3.17. Very totally irregular MFG

Let G be a connected MFG. If atleast one vertex of G has distinct very closed neighbors degree, then G is a very totally irregular MFG.

Definition 3.18. Highly very totally irregular MFG

Let G be a connected MFG. If every vertex of G have distinct very closed neighbors degrees, then G is highly very totally irregular MFG.

Definition 3.19. mem-measures

Let G be a connected MFG then

i) The mem-length of the path $l_{mem}(p) = \sum_{i=1}^{n-1} \mu(v_i, v_{i+1})$,

where p : $v_1, \ldots, v_n$

ii) mem-distance $\delta_{mem}(v_i, v_j) = \min(l_{mem}(p))$ iii) mem-eccentricity $e_{mem}(v_i) = \max\{\delta_{mem}(v_i, v_j) : v_i \in V, v_i \neq v_j\}$ iv) mem-radius of G is

$r_{mem}(G) = \min\{e_{mem}(v_i) : v_i \in V\}$

v) mem-diameter of G is $d_{mem}(G) = \max\{_{mem}(v_i) : v_i \in V\}$

Definition 3.19. Self centered MFG

Let G be a connected MFG then G is said to be self centered MFG if $r_{mem}(G) = d_{mem}(G)$.

Example 6.

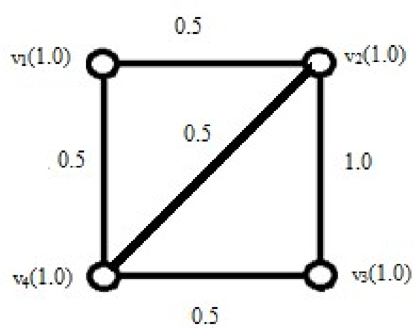


Fig.9: G

$l_{mem}(v_1v_2) = 0.5$, $l_{mem}(v_1v_3v_2) = 1.5$, $l_{mem}(v_1v_4v_3v_2) = 2.0$, $l_{mem}(v_1v_4v_3v_1v_2) = 2.0$ $\delta mem(v_1,v_2)$

$= \min(l_{mem}(v_1v_2), l_{mem}(v_1v_3v_2), l_{mem}(v_1v_4v_3v_2), l_{mem}(v_1v_4v_3v_1v_2)) = 0.5$

Similarly, $\delta_{mem}(v_1,v_3) = 0.5$, $\delta_{mem}(v_1,v_4) = 0.5$, $\delta_{mem}(v_2,v_3) = 1.0$, $\delta_{mem}(v_2,v_4) = 1.0$ and $\delta mem(v_3,v_4)$ $= 0.5$ $e mem(v_1) = 0.5$, $e_{mem}(v_2) = 1.0$, $e_{mem}(v_3) = 1.0$ and $e_{mem}(v_4) = 1.0$. $r_{mem}(G) = 0.5$ and $d_{mem}(G)$ $= 1.0$.

Example 7.

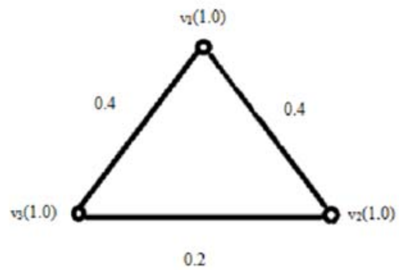


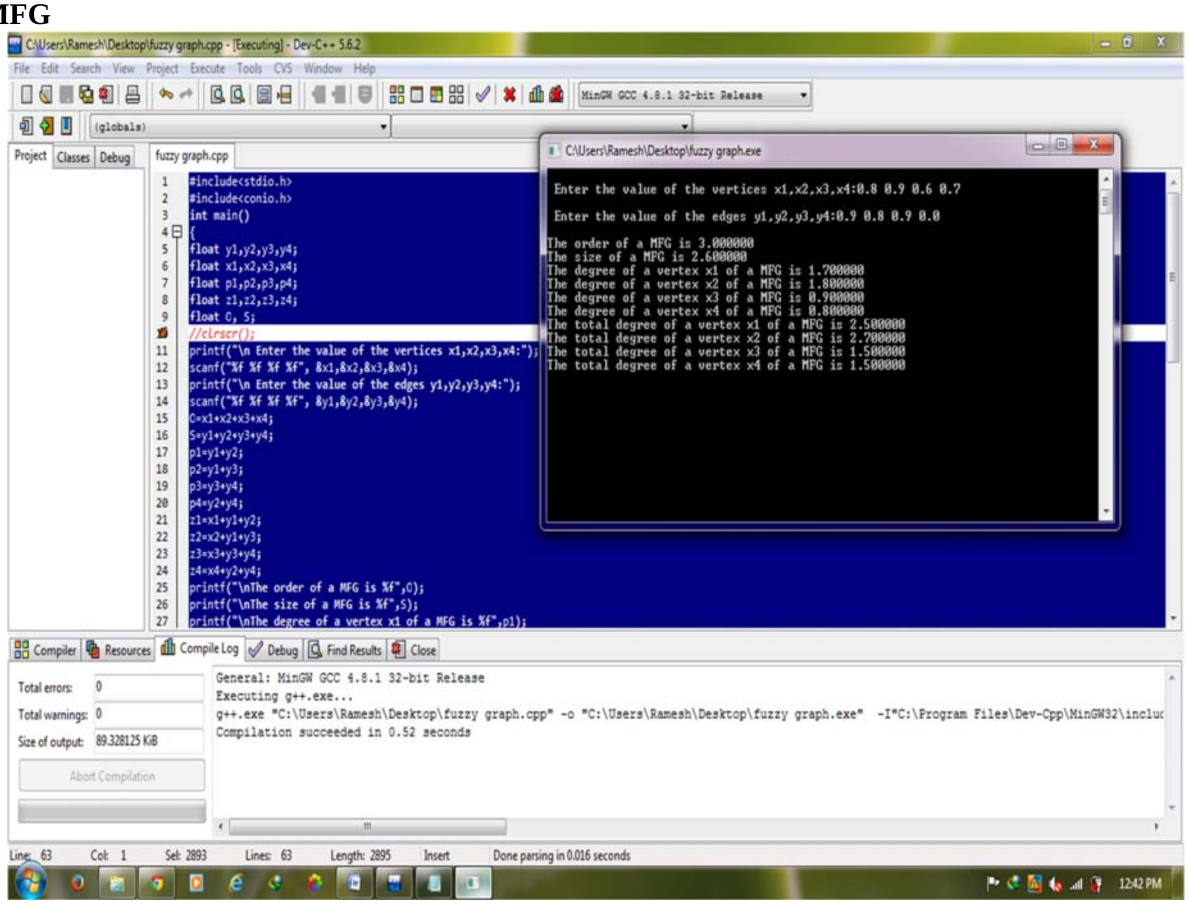
Fig.10: G

i) Mem-distance is $\delta_{mem}(v_1,v_2) = 0.4$, $\delta_{mem}(v_1,v_3) = 0.4$, $\delta_{mem}(v_2,v_3) = 0.2$.

ii) Mem-eccentricity of each vertex is 0.4.

iii) Mem-Radius of G is 0.4 and Mem-diameter of G is 0.4.
$r_{mem}(G) = d_{mem}(G)$. Hence G is self centered graph.

Simulation result using C program to find the order, size, degree and total degree of a MFG



4. Conclusion

Graph theory has various applications in different fields of science. Since many real world complex systems can be modeled using graphs, many researchers developed some properties related to the definition introduced for fuzzy graph by Azirel Rosenfeld [7] and anti fuzzy graph by **Akram** [1]. As a continuation, in this paper we have introduced few notions related to modified fuzzy graph and modified anti fuzzy graph introduced by [2].

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