RELIABLE H_∞ CONTROL DESIGN FOR UNCERTAIN NEUTRAL TYPE SYSTEM WITH TIME-VARYING DELAYS

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Abstract
This paper is concerned with the reliable H_∞ for a class of uncertain neutral system with time- varying delays. A new Lyapunov functional is constructed to obtain sufficient conditions under which the uncertain neutral system is (6) with disturbance attenuation level $\gamma > 0$ for all admissible uncertainties. More precisely, Schur complement and Jenson integral inequality is are utilized to substantially simplify the derivation of the main result. Finally, a numerical example with simulation result is provided to show the effectiveness of the obtained result.

Index Terms: Reliable H_∞ control, uncertain neutral system, parameter uncertainties.

Notations: Throughout this paper, Superscripts "T" and "(-1)" stand for matrix transposition and matrix inverse respectively. $\mathfrak{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{Z}_+$ denotes the set of positive integers. $\mathfrak{R}^{n\,xn}$ denotes the set of all nxn real matrices. $P > 0$ (respectively $P < 0$) means that P is positive definite (respectively negative definite). I and 0 represent identity matrix and zero matrix with compatible dimension. * denotes the symmetric elements of the symmetric matrix.

I. INTRODUCTION

During the past few decades, much attention has been paid to the research on the problem of time delay, which frequently occurs in various practical engineering systems, such as T-S fuzzy system, switched linear system, Markovian jump system and networked control system. The existence of time delay would deteriorate the performance of system or even be the important source of instability of systems with time-varying delay is frequently has attracted remarkable attention of researchers, see for instance in. The stability analysis criteria for time-delay systems can be classified into two types: delay-independent ones and delay-dependent ones.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider an uncertain neutral system with time varying delays in the following form

$$\begin{cases} \dot{x}(t) - \bar{C}\dot{x}\big(t-\tau(t)\big) = \bar{A}x(t) + \overline{A_d}x\big(t-h(t)\big) + Bu_f(t) + \omega(t) \\ y(t) = D\,x(t) + D_d\,x\big(t-h(t)\big), \end{cases} \tag{1}$$

where $x(t) \in \mathfrak{R}^n$ is the state vector. $\bar{A} = A + \Delta A(t), \overline{A_d} = A_d + \Delta A_d(t),\ \bar{C} = C + \Delta C(t)$, where $A, A_d, C, D \in \mathfrak{R}^{nXn}$ and $B \in \mathfrak{R}^{mXn}$ are known real constant matrices with appropriate dimensions. $\tau(t)$ and $h(t)$ are time-varying delays satisfying

$$0 \le \tau_1 \le \tau(t) \le \tau_2, 0 \le h_1 \le h(t) \le h_2, \dot{\tau}_1(t) = \mu_1 \text{ and } \dot{h}(t) = \mu_2, \tag{2}$$

where $\tau_1, \tau_2, h_1, h_2, \mu_1$ and μ_2 are positive constants. The parameter uncertainties $\Delta A(t), \Delta A_d(t)$ and $\Delta C(t)$ are time-varying matrices with appropriate dimensions and are assumed to be norm bounded and are defined as

$$[\Delta C(t)\ \ \Delta A(t)\ \ \Delta A_d(t)] = M\,F(t)\,[N_1\ \ N_2\ \ N_3], \tag{3}$$

where M, N_1, N_2 ,and N_3 are known constant real matrices with appropriate dimensions. $F(t)$ is the uncertain matrix functions which satisfies

$$F^T(t)\,F(t) \le I. \tag{4}$$

The control input can be described as

$$u_f(t) = H\,u(t) = H\,K\,x(t), \tag{5}$$

where K is the feedback gain matrix and H is fault matrix.
Finally, by combining (1) and (5) we obtain the closed-loop uncertain neural system with time varying delays in the following form

$$\begin{cases} \dot{x}(t) - \bar{C}\dot{x}(t-\tau(t)) = \bar{A}x(t) + \overline{A_d}x(t-h(t)) + BH\,K\,x(t) + \omega(t) \\ y(t) = D\,x(t) + D_d\,x(t-h(t)), \end{cases} \tag{6}$$

Definition 2.1: [27] System (6) is said to be robustly stable with disturbance attenuation γ if for all $\omega(t) \in L_2[0,\infty]$, the response $z(t)$ under zero initial conditions satisfies
$\int_0^\infty z^T(t)\, z(t) dt \le \gamma^2 \int_0^\infty \omega^T(t)\, \omega(t) dt$.

Lemma 2.2: [20] Let D, E and $F(k)$ be the real matrices of appropriate dimensions with $F(k)$ satisfying $F^T(k)\, F(k) \le I$. Then we have the following inequality holds:
for $\epsilon > 0, D\, F(k)\, E + E^T\, F^T(k)\, D^T \le \epsilon^{-1}\, D\, D^T + \epsilon\, E^T E$.

Lemma 2.3: [20] Given constant matrices Ξ_1, Ξ_2 and Ξ_3 appropriate dimensions, where $\Xi_1 = \Xi_1^T > 0$ and $\Xi_2 = \Xi_2^T > 0$ then $\Xi_1 + \Xi_3^T\, \Xi_2^{-1}\, \Xi_3 < 0$ if and only if $\begin{bmatrix} \Xi_1 & \Xi_3^T \\ * & -\Xi_2 \end{bmatrix} < 0$.

Lemma 24: [12] For any constant matrices $M > 0$, any scalars a and b with $a < b$, and a vector function $x(t)\colon [a,b] \to \Re^n$ such that the integrals concerned are all defined, then the following holds:

$$\left[\int_a^b x(s)\, ds\right]^T M \left[\int_a^b x(s)\, ds\right] \le (b-a) \int_a^b x^T(s)\, M\, x(s) ds$$

III. MAIN RESULTS

The main aim of this section is to obtain the conditions for the existence of a stabilizing state feedback reliable H_∞ control law such that the resulting closed-loop system is robustly stable with given disturbance attenuation level γ > 0. In order to discuss robust stability of (6) which has parameter uncertainties, first we consider the case in which the matrices are fixed, i.e., when $\Delta C(t) = 0$, $\Delta A(t) = 0$ and $\Delta A_d(t) = 0$. For this, we consider the nominal form of system as follows:

$$\begin{cases} \dot{x}(t) - C\,\dot{x}(t-\tau(t)) = A\,x(t) + A_d\,x(t-h(t)) + Bu_f(t) + \omega(t) \\ y(t) = D\,x(t) + D_d\,x(t-h(t)), \end{cases} \tag{7}$$

Theorem 3.1: For given positive scalars $\tau_1, \tau_2, h_1, h_2, \mu_1$ and μ_2 and the known actuator fault matrix H there exists a reliable controller (5) such that the nominal neutral system (7) satisfies H_∞ performance index γ if there exists symmetric matrices $P > 0, Q_i > 0, i = 1,2,\dots,9, R_i > 0, S_i > 0, i = 1,2,3,4$ with appropriate dimensions and any matrices X and Y such that the following matrix inequality holds: $[\widehat{\Omega}_{17x17}] < 0$, where

$\widehat{\Omega}_{1,1} = \hat{Q}_1 + \hat{Q}_2 + \hat{Q}_4 + \hat{Q}_5 - 2\hat{S}_1 - 2\hat{S}_3 + h_1\hat{R}_1 + \tau_1\hat{R}_3 - 2X, \widehat{\Omega}_{1,8} = \hat{P} + AX + BHY,$

$\widehat{\Omega}_{1,12} = \frac{2}{h_1}\hat{S}_1, \widehat{\Omega}_{1,14} = \frac{2}{\tau_1}\hat{S}_3, \widehat{\Omega}_{2,2} = \hat{Q}_3 - \hat{Q}_1 + (h_2 - h_1)\,\hat{R}_2 - 2\hat{S}_2, \widehat{\Omega}_{2,13} = \frac{2}{h_2 - h_1}\hat{S}_2,$

$\widehat{\Omega}_{3,3} = -\hat{Q}_3, \widehat{\Omega}_{4,4} = \hat{Q}_6 - \hat{Q}_4 - 2\hat{S}_4, \widehat{\Omega}_{4,15} = \frac{2}{\tau_2 - \tau_1}\hat{S}_4, \widehat{\Omega}_{5,5} = -\hat{Q}_6, \widehat{\Omega}_{6,6} = -(1-\mu_1)\hat{Q}_2,$

$\widehat{\Omega}_{6,8} = \overline{A_d}\,X, \widehat{\Omega}_{7,7} = -(1-\mu_2)\hat{Q}_5, \widehat{\Omega}_{8,8}\hat{Q}_7 + \frac{h_1^2}{2}\hat{S}_1 + \frac{\tau_1^2}{2}\hat{S}_3 + \hat{Q}_8 + \hat{Q}_9 - 2X, \widehat{\Omega}_{8,9} = \bar{C}\,X,$

$\widehat{\Omega}_{8,16} = X, \widehat{\Omega}_{9,9} = -(1-\mu_2)\hat{Q}_7, \widehat{\Omega}_{10,10} = \frac{(h_2 - h_1)^2}{2}\hat{S}_2 - \hat{Q}_8, \widehat{\Omega}_{11,11} = \frac{(\tau_2 - \tau_1)^2}{2}\hat{S}_4 - \hat{Q}_9,$

$\widehat{\Omega}_{12,12} = -\frac{2}{h_1^2}\hat{S}_1 - \frac{1}{h_1}\hat{R}_1, \widehat{\Omega}_{13,13} = -\frac{2}{(h_2 - h_1)^2}\hat{S}_2 - \frac{1}{(h_2 - h_1)}\hat{R}_2, \widehat{\Omega}_{14,14} = -\frac{2}{\tau_1^2}\hat{S}_3 - \frac{1}{\tau_1}\hat{R}_3,$

$\widehat{\Omega}_{14,14} = -\frac{2}{\tau_1^2}\hat{S}_3 - \frac{1}{\tau_1}\hat{R}_3, \widehat{\Omega}_{15,15} = -\frac{2}{(\tau_2 - \tau_1)^2}\hat{S}_4 - \frac{1}{(\tau_2 - \tau_1)}\hat{R}_4, \widehat{\Omega}_{16,16} = -\gamma^2, \widehat{\Omega}_{1,17} = XD^T,$

$\widehat{\Omega}_{6,17} = XD_d^T, \widehat{\Omega}_{17,17} = -I$, where $\hat{P} = W^{-1}P\,W^{-1}, \hat{Q}_i = W^{-1}Q_i\,W^{-1}, i = 1,2,3 \dots, 9$

$\hat{R}_i = W^{-1}R_i W^{-1}, \hat{S}_i = W^{-1}S_i W^{-1}, i = 1,2,3,4.$

Proof: Let us define the Lyapunov – Krasovskii function for the system (7)

$$V(x(t)) = \sum_{m=1}^{4} V_m(x(t)), \tag{8}$$

where

$$V_1(x(t)) = x^T(t)P\, x(t),$$

$$\begin{aligned} V_2(x(t)) = &\int_{t-h_1}^{t} x^T(\theta)\, Q_1\, x(\theta)d\theta + \int_{t-h(t)}^{t} x^T(\theta)\, Q_2\, x(\theta)d\theta + \int_{t-h_2}^{t-h_1} x^T(\theta)\, Q_3\, x(\theta)d\theta \\ &+ \int_{t-\tau_1}^{t} x^T(\theta)\, Q_4\, x(\theta)d\theta + \int_{t-\tau(t)}^{t} x^T(\theta)\, Q_5\, x(\theta)d\theta + \int_{t-\tau_2}^{t-\tau_1} x^T(\theta)\, Q_6\, x(\theta)d\theta \\ &+ \int_{t-\tau(t)}^{t} \dot{x}^T(\theta)\, Q_7\, \dot{x}(\theta)d\theta + \int_{t-h_1}^{t} \dot{x}^T(\theta)\, Q_8\, \dot{x}(\theta)d\theta + \int_{t-\tau_1}^{t} \dot{x}^T(\theta)\, Q_9\, \dot{x}(\theta)d\theta \end{aligned}$$

$$\begin{aligned} V_3(x(t)) = &\int_{t-h_1}^{t}\int_{u}^{t} x^T(\theta)\, R_1\, x(\theta)d\theta\, du + \int_{t-h_2}^{t-h_1}\int_{u}^{t-h_1} x^T(\theta)\, R_2\, x(\theta)d\theta\, du \\ &+ \int_{t-\tau_1}^{t}\int_{u}^{t} x^T(\theta)\, R_3\, x(\theta)d\theta\, du + \int_{t-\tau_2}^{t-\tau_1}\int_{u}^{t} x^T(\theta)\, R_4\, x(\theta)d\theta\, du \end{aligned}$$

$$\begin{aligned} V_4(x(t)) = &\int_{t-h_1}^{t}\int_{u}^{t}\int_{\lambda}^{t} \dot{x}^T(\theta)\, S_1\, \dot{x}(\theta)d\theta\, d\lambda\, du + \int_{t-h_2}^{t-h_1}\int_{u}^{t-h_1}\int_{\lambda}^{t-h_1} \dot{x}^T(\theta)\, S_2\, \dot{x}(\theta)d\theta\, d\lambda\, du \\ &+ \int_{t-\tau_1}^{t}\int_{u}^{t}\int_{\lambda}^{t} \dot{x}^T(\theta)\, S_3\, \dot{x}(\theta)d\theta\, d\lambda\, du + \int_{t-\tau_2}^{t-\tau_1}\int_{u}^{t-\tau_1}\int_{\lambda}^{t-\tau_1} \dot{x}^T(\theta)\, S_4\, \dot{x}(\theta)d\theta\, d\lambda\, du \end{aligned}$$

Calculating the derivatives $\dot{V}(x(t))$ along the trajectories of the system (7), we have

$$\dot{V}_1(x(t)) = 2\, x^T(t)\, P\, \dot{x}(t), \tag{9}$$

$$\begin{aligned} \dot{V}_2(x(t)) = &\ x^T(t)[Q_! + Q_2 + Q_4 + Q_5]x(t) + x^T(t-h_1)[Q_3 - Q_1]x(t-h_1) \\ &+ x^T(t-h_2)[-Q_3]x(t-h_2) + x^T(t-\tau_1)[Q_6 - Q_4]x(t-\tau_1) \\ &+ x^T(t-\tau_2)[-Q_6]x(t-\tau_2) + x^T(t-h(t))[-(1-\mu_1)Q_2]x(t-h(t)) \\ &+ x^T(t-\tau(t))[-(1-\mu_2)Q_5]x(t-\tau(t)) + \dot{x}^T(t)[Q_7 + Q_8 + Q_9]\dot{x}(t) \\ &+ \dot{x}^T(t-\tau(t))[-(1-\mu_2)Q_7]\dot{x}(t-\tau(t)) + \dot{x}^T(t-h_1)[-Q_8]\dot{x}(t-h_1) \\ &+ \dot{x}^T(t-\tau_1)[-Q_9]\dot{x}(t-\tau_1) \end{aligned} \tag{10}$$

$$\dot{V}_3(x(t)) = h_1 x^T(t)\, R_1\, x(t) - \int_{t-h_1}^{t} x^T(s)\, R_1\, x(s)ds + (h_2 - h_1)\, x^T(t-h_1)\, R_2\, x(t-h_1)$$
$$- \int_{t-h_2}^{t-h_1} x^T(s)\, R_2\, x(s)ds + \tau_1 x^T(t)\, R_3\, x(t) - \int_{t-\tau_1}^{t} x^T(s)\, R_3\, x(s)ds$$
$$+ (\tau_2 - \tau_1) x^T(t-\tau_1) R_4 x(t-\tau_1)$$
$$- \int_{t-\tau_2}^{t-\tau_1} x^T(s)\, R_4\, x(s)ds \qquad (11)$$

$$\dot{V}_4(x(t)) = \frac{h_1^2}{2}\dot{x}^T(t)\, S_1\, \dot{x}(t) - \int_{t-h_1}^{t}\int_{\lambda}^{t} \dot{x}^T(\theta)\, S_1\, \dot{x}(\theta)d\theta\, d\lambda + \frac{(h_2-h_1)^2}{2}\dot{x}^T(t-h_1)S_2\dot{x}(t-h_1)$$
$$- \int_{t-h_2}^{t-h_1}\int_{\lambda}^{t-h_1} \dot{x}^T(\theta)\, S_2\, \dot{x}(\theta)d\theta\, d\lambda + \frac{\tau_1^2}{2}\dot{x}^T(t)\, S_3\, \dot{x}(t) - \int_{t-\tau_1}^{t}\int_{\lambda}^{t} \dot{x}^T(\theta)\, S_3\, \dot{x}(\theta)d\theta\, d\lambda +$$
$$+ \frac{(\tau_2-\tau_1)^2}{2}\dot{x}^T(t-\tau_1)S_4\dot{x}(t-\tau_1) - \int_{t-\tau_2}^{t-\tau_1}\int_{\lambda}^{t} \dot{x}^T(\theta)\, S_4\, \dot{x}(\theta)d\theta\, d\lambda \qquad (12)$$

By applying Jenson inequality Lemma 2.4 for the integral terms in (11), (12) and the time delay interval, the integration in the above equations can be written as

$$- \int_{t-h_1}^{t} x^T(s)\, R_1\, x(s)ds$$
$$\leq -\frac{1}{h_1}\left[\int_{t-h_1}^{t} x(s)ds\right] R_1 \left[\int_{t-h_1}^{t} x(s)ds\right] \qquad (13)$$

$$- \int_{t-h_2}^{t-h_1} x^T(s)\, R_2\, x(s)ds$$
$$\leq -\frac{1}{h_2 - h_1}\left[\int_{t-h_2}^{t-h_1} x(s)ds\right]^T R_2 \left[\int_{t-h_2}^{t-h_1} x(s)ds\right] \qquad (14)$$

$$- \int_{t-\tau_1}^{t} x^T(s)\, R_3\, x(s)ds$$
$$\leq -\frac{1}{\tau_1}\left[\int_{t-\tau_1}^{t} x(s)ds\right] R_3 \left[\int_{t-\tau_1}^{t} x(s)ds\right] \qquad (15)$$

$$- \int_{t-\tau_2}^{t-\tau_1} x^T(s)\, R_4\, x(s)ds$$
$$\leq -\frac{1}{\tau_2 - \tau_1}\left[\int_{t-\tau_2}^{t-\tau_1} x(s)ds\right]^T R_4 \left[\int_{t-\tau_2}^{t-\tau_1} x(s)ds\right] \qquad (16)$$

$$-\int_{t-h_1}^{t}\int_{\lambda}^{t}\dot{x}^T(\theta)\,S_1\,\dot{x}(\theta)d\theta\,d\lambda$$

$$\leq -\frac{2}{h_1^2}\left[\int_{t-h_1}^{t}\int_{\lambda}^{t}\dot{x}(\theta)d\theta\,d\lambda\right]S_1\left[\int_{t-h_1}^{t}\int_{\lambda}^{t}\dot{x}(\theta)d\theta\,d\lambda\right] \quad (17)$$

$$-\int_{t-h_2}^{t-h_1}\int_{\lambda}^{t-h_1}\dot{x}^T(\theta)\,S_2\,\dot{x}(\theta)d\theta\,d\lambda$$

$$\leq -\frac{2}{(h_2-h_1)^2}\left[\int_{t-h_2}^{t-h_1}\int_{\lambda}^{t-h_1}\dot{x}(\theta)d\theta\,d\lambda\right]^T S_2\left[\int_{t-h_2}^{t-h_1}\int_{\lambda}^{t-h_1}\dot{x}(\theta)d\theta\,d\lambda\right] \quad (18)$$

$$-\int_{t-\tau_1}^{t}\int_{\lambda}^{t}\dot{x}^T(\theta)\,S_3\,\dot{x}(\theta)d\theta\,d\lambda$$

$$\leq -\frac{2}{\tau_1^2}\left[\int_{t-\tau_1}^{t}\int_{\lambda}^{t}\dot{x}(\theta)d\theta\,d\lambda\right]S_3\left[\int_{t-\tau_1}^{t}\int_{\lambda}^{t}\dot{x}(\theta)d\theta\,d\lambda\right] \quad (19)$$

$$-\int_{t-\tau_2}^{t-\tau_1}\int_{\lambda}^{t}\dot{x}^T(\theta)\,S_4\,\dot{x}(\theta)d\theta\,d\lambda$$

$$\leq -\frac{2}{(\tau_2-\tau_1)^2}\left[\int_{t-\tau_2}^{t-\tau_1}\int_{\lambda}^{t}\dot{x}(\theta)d\theta\,d\lambda\right]^T S_4\left[\int_{t-\tau_2}^{t-\tau_1}\int_{\lambda}^{t}\dot{x}(\theta)d\theta\,d\lambda\right] \quad (20)$$

At the same time, for any appropriate dimensional non-singular matrix W and a scalar $\gamma > 0$, we have $2\,\dot{x}^T(t)\,W\left[C\,\dot{x}(t-\tau(t)) + A\,x(t) + A_d\,x(t-h(t)) + BHK\,x(t) + \omega(t) - \dot{x}(t)\right] = 0$
It follows from $(9)-(20)$ that

$$\begin{aligned}\dot{V}(x(t)) \leq\ & 2\,x^T(t)\,P\,\dot{x}(t) + x^T(t)[Q_! + Q_2 + Q_4 + Q_5]x(t) + x^T(t-h_1)[Q_3 - Q_1]x(t-h_1)\\ &+ x^T(t-h_2)[-Q_3]x(t-h_2) + x^T(t-\tau_1)[Q_6 - Q_4]x(t-\tau_1)\\ &+ x^T(t-\tau_2)[-Q_6]x(t-\tau_2) + x^T(t-h(t))[-(1-\mu_1)Q_2]x(t-h(t))\\ &+ x^T(t-\tau(t))[-(1-\mu_2)Q_5]x(t-\tau(t)) + \dot{x}^T(t)[Q_7 + Q_8 + Q_9]\dot{x}(t)\\ &+ \dot{x}^T(t-\tau(t))[-(1-\mu_2)Q_7]\dot{x}(t-\tau(t)) + \dot{x}^T(t-h_1)[-Q_8]\dot{x}(t-h_1)\\ &+ \dot{x}^T(t-\tau_1)[-Q_9]\dot{x}(t-\tau_1) + h_1x^T(t)\,R_1\,x(t)\\ &+ (h_2-h_1)\,x^T(t-h_1)\,R_2\,x(t-h_!) + \tau_1x^T(t)\,R_3\,x(t)\\ &+ (\tau_2-\tau_1)x^T(t-\tau_1)R_4x(t-\tau_1) + \frac{h_1^2}{2}\dot{x}^T(t)\,S_1\,\dot{x}(t)\\ &+ \frac{(h_2-h_1)^2}{2}\dot{x}^T(t-h_1)S_2\dot{x}(t-h_1) + \frac{\tau_1^2}{2}\dot{x}^T(t)\,S_3\,\dot{x}(t)\end{aligned}$$

$$+\frac{(\tau_2-\tau_1)^2}{2}\dot{x}^T(t-\tau_1)S_4\dot{x}(t-\tau_1)-\frac{1}{h_1}\left[\int_{t-h_!}^{t}x(s)ds\right]R_1\left[\int_{t-h_!}^{t}x(s)ds\right]$$

$$-\frac{1}{h_2-h_1}\left[\int_{t-h_2}^{t-h_1}x(s)ds\right]^T R_2\left[\int_{t-h_2}^{t-h_1}x(s)ds\right]-\frac{1}{\tau_1}\left[\int_{t-\tau_1}^{t}x(s)ds\right]^T R_3\left[\int_{t-\tau_1}^{t}x(s)ds\right]$$

$$-\frac{1}{\tau_2-\tau_1}\left[\int_{t-\tau_2}^{t-\tau_1}x(s)ds\right]^T R_4\left[\int_{t-\tau_2}^{t-\tau_1}x(s)ds\right]-\frac{2}{h_1^2}\left[\int_{t-h_1}^{t}\int_{\lambda}^{t}\dot{x}(\theta)d\theta\, d\lambda\right]^T S_1\left[\int_{t-h_1}^{t}\int_{\lambda}^{t}\dot{x}(\theta)d\theta\, d\lambda\right]$$

$$-\frac{2}{(h_2-h_1)^2}\left[\int_{t-h_2}^{t-h_1}\int_{\lambda}^{t-h_1}\dot{x}(\theta)d\theta\, d\lambda\right]^T S_2\left[\int_{t-h_2}^{t-h_1}\int_{\lambda}^{t-h_1}\dot{x}(\theta)d\theta\, d\lambda\right]$$

$$-\frac{2}{\tau_1^2}\left[\int_{t-\tau_1}^{t}\int_{\lambda}^{t}\dot{x}(\theta)d\theta\, d\lambda\right]S_3\left[\int_{t-\tau_1}^{t}\int_{\lambda}^{t}\dot{x}(\theta)d\theta\, d\lambda\right]$$

$$-\frac{2}{(\tau_2-\tau_1)^2}\left[\int_{t-\tau_2}^{t-\tau_1}\int_{\lambda}^{t}\dot{x}(\theta)d\theta\, d\lambda\right]^T S_4\left[\int_{t-\tau_2}^{t-\tau_1}\int_{\lambda}^{t}\dot{x}(\theta)d\theta\, d\lambda\right] \qquad (21)$$

To discuss the H_∞ performance of system (7), we introduce the following relation:

$$J_n=\int_0^\infty [y^T(t)\,y(t) - \gamma^2\,\omega^T(t)w(t)]\,dt \qquad (22)$$

It follows from (21) using Definition 2.1 and Lemma 2.2, by the zero initial condition, we have $V(0)=0$ and $V(\infty)\geq 0$, and we have

$$J_n \leq \int_0^\infty \left[y^T(t)\,y(t)-\gamma^2\,\omega^T(t)w(t)+\dot{V}\big(x(t)\big)\right]dt$$

$$\leq \int_0^\infty \varsigma^T(t)\,\Pi_1\,\varsigma(t)\,dt \qquad (23)$$

where,

$$\varsigma^T(t)=[x^T(t)\quad x^T(t-h_1)\quad x^T(t-h_2)\quad x^T(t-\tau_1)\quad x^T(t-\tau_2)\quad x^T\big(t-h(t)\big)\quad x^T\big(t-\tau(t)\big)\quad \dot{x}^T(t)\quad \dot{x}^T\big(t-\tau(t)\big)\quad \dot{x}^T(t-h_1)\quad \dot{x}^T(t-\tau_1)$$
$$\int_{t-h_!}^{t}x^T(s)ds\quad \int_{t-h_2}^{t-h_1}x^T(s)ds\quad \int_{t-\tau_1}^{t}x^T(s)ds\quad \int_{t-\tau_1}^{t}x^T(s)ds\quad \int_{t-\tau_2}^{t-\tau_1}x^T(s)ds\quad \omega^T(t)]$$

and $\Pi_1=[\Omega_{17x17}]<0$

where, $\Omega_{1,1}=Q_1+Q_2+Q_4+Q_5-2S_1-2S_3+h_1R_1+\tau_1R_3-2X$, $\Omega_{1,8}=P+AX+BHY$, $\Omega_{1,12}=\frac{2}{h_1}S_1$, $\Omega_{1,14}=\frac{2}{\tau_1}S_3$, $\Omega_{2,2}=Q_3-Q_1+(h_2-h_1)R_2-2S_2$, $\Omega_{2,13}=\frac{2}{h_2-h_1}S_2$, $\Omega_{3,3}=-Q_3$, $\Omega_{4,4}=Q_6-Q_4-2S_4$, $\Omega_{4,15}=\frac{2}{\tau_2-\tau_1}S_4$, $\Omega_{5,5}=-Q_6$, $\Omega_{6,6}=-(1-\mu_1)Q_2$, $\Omega_{6,8}=A_dX$, $\Omega_{7,7}=-(1-\mu_2)Q_5$, $\Omega_{8,8}=Q_7+Q_8+Q_9-2X+\frac{h_1^2}{2}S_!+\frac{\tau_1^2}{2}S_3$, $\Omega_{8,9}=CX$, $\Omega_{8,16}=X$, $\Omega_{9,9}=-(1-\mu_2)Q_7$, $\Omega_{10,10}=\frac{(h_2-h_1)^2}{2}S_2-Q_8$, $\Omega_{11,11}=\frac{(\tau_2-\tau_1)^2}{2}S_4-Q_9$, $\Omega_{12,12}=-\frac{2}{h_1^2}S_1-\frac{1}{h_1}R_1$, $\Omega_{13,13}=-\frac{2}{(h_2-h_1)^2}S_2-\frac{1}{h_2-h_1}R_2$, $\Omega_{14,14}=-\frac{2}{\tau_1^2}S_3-\frac{1}{\tau_!}R_3$,

$\Omega_{15,15} = -\frac{2}{(\tau_2-\tau_1)^2}S_4 - \frac{1}{\tau_2-\tau_1}R_4,\ \Omega_{15,15} = -\gamma^2,\ \Omega_{1,17} = XD^T,\ \Omega_{6,17} = XD_d^T,\ \Omega_{17,17} = -I.$

To complete the proof , pre and post multiplying (19) by $diag\{W^{-1},\dots,W^{-1},I\}$ and by letting $\hat{P} = W^{-1}P\,W^{-1}, \hat{Q}_i = W^{-1}Q_i\,W^{-1}, i = 1,2,\dots,9, \hat{R}_i = W^{-1}R_i\,W^{-1}, \hat{S}_i = W^{-1}S_i\,W^{-1}, i = 1,2,3,4$

The following matrix holds:

$[\widehat{\Omega}_{17x17}] < 0$

where,

$\widehat{\Omega}_{1,1} = \hat{Q}_1 + \hat{Q}_2 + \hat{Q}_4 + \hat{Q}_5 - 2\hat{S}_1 - 2\hat{S}_3 + h_1\hat{R}_1 + \tau_1\hat{R}_3 - 2X, \quad \widehat{\Omega}_{1,8} = \hat{P} + AX + BHY,$

$\widehat{\Omega}_{1,12} = \frac{2}{h_1}\hat{S}_1, \quad \widehat{\Omega}_{1,14} = \frac{2}{\tau_1}\hat{S}_3, \quad \widehat{\Omega}_{2,2} = \hat{Q}_3 - \hat{Q}_1 + (h_2-h_1)\,\hat{R}_2 - 2\hat{S}_2,\ \widehat{\Omega}_{2,13} = \frac{2}{h_2-h_1}\hat{S}_2,$

$\widehat{\Omega}_{3,3} = -\hat{Q}_3,\ \widehat{\Omega}_{4,4} = \hat{Q}_6 - \hat{Q}_4 - 2\hat{S}_4,\ \widehat{\Omega}_{4,15} = \frac{2}{\tau_2-\tau_1}\hat{S}_4,\ \widehat{\Omega}_{5,5} = -\hat{Q}_6,\ \widehat{\Omega}_{6,6} = -(1-\mu_1)\hat{Q}_2,$

$\widehat{\Omega}_{6,8} = \overline{A_d}\,X, \widehat{\Omega}_{7,7} = -(1-\mu_2)\hat{Q}_5, \widehat{\Omega}_{8,8} = \hat{Q}_7 + \frac{h_1^2}{2}\hat{S}_1 + \frac{\tau_1^2}{2}\hat{S}_3 + \hat{Q}_8 + \hat{Q}_9 - 2X, \widehat{\Omega}_{8,9} = \bar{C}\,X,$

$\widehat{\Omega}_{8,16} = X, \widehat{\Omega}_{9,9} = -(1-\mu_2)\hat{Q}_7, \widehat{\Omega}_{10,10} = \frac{(h_2-h_1)^2}{2}\hat{S}_2 - \hat{Q}_8, \widehat{\Omega}_{11,11} = \frac{(\tau_2-\tau_1)^2}{2}\hat{S}_4 - \hat{Q}_9,$

$\widehat{\Omega}_{12,12} = -\frac{2}{h_1^2}\hat{S}_1 - \frac{1}{h_1}\hat{R}_1, \widehat{\Omega}_{13,13} = -\frac{2}{(h_2-h_1)^2}\hat{S}_2 - \frac{1}{(h_2-h_1)}\hat{R}_2, \widehat{\Omega}_{14,14} = -\frac{2}{\tau_1^2}\hat{S}_3 - \frac{1}{\tau_1}\hat{R}_3,$

$\widehat{\Omega}_{14,14} = -\frac{2}{\tau_1^2}\hat{S}_3 - \frac{1}{\tau_1}\hat{R}_3, \widehat{\Omega}_{15,15} = -\frac{2}{(\tau_2-\tau_1)^2}\hat{S}_4 - \frac{1}{(\tau_2-\tau_1)}\hat{R}_4, \widehat{\Omega}_{16,16} = -\gamma^2, \widehat{\Omega}_{1,17} = XD^T,$

$\widehat{\Omega}_{6,17} = XD_d^T, \quad \widehat{\Omega}_{17,17} = -I.$

Theorem 3.2: For given positive scalars $\tau_1, \tau_2, h_1, h_2, \mu_1$ and μ_2 and the known actuator fault matrix H and M, N_1, N_2 and N_3 are known constant real matrices with appropriate dimensions there exists a reliable controller (5) such that the nominal neutral system (7) satisfies H_∞ performance index γ if there exists symmetric matrices $P > 0, Q_i > 0, i = 1,2,\dots,9, R_i > 0, S_i > 0, i = 1,2,3,4$ with appropriate dimensions and any matrices X and Y such that the following matrix inequality holds:

$$\widehat{\Psi} = \begin{bmatrix} [\widehat{\Omega}_{l,m}]_{17x17} & \widehat{\Omega}_1 & \widehat{\Omega}_2 \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0$$

where $\widehat{\Omega}_1 = [N_1X \quad 0_{4n} \quad N_2X \quad 0_{2n} \quad N_3X]^T, \widehat{\Omega}_2 = [\varepsilon M^T \quad 0_{4n} \quad \varepsilon M^T \quad 0_{2n} \quad \varepsilon M^T]^T$

In this case, state feedback control gain in (5) is given by $K = Y\,X^{-1}$ and the other parameters are defined as in Theorem 3.1.

Proof: The proof of this theorem is immediately follows from Theorem 3.1 by replacing the matrices A, A_d and C with $A + M\,\Delta(t)\,N_1$, $A_d + M\,\Delta(t)\,N_2$ and $C + M\,\Delta(t)\,N_3$ respectively. Further, by applying Lemma 2.1 and Lemma 2.3 we can obtain (23). This implies that the H_∞ performance can be ensured if the matrix inequality in Theorem 3.2 hold. The proof is completed.

IV. NUMERICAL SIMULATION

In this section, we provide a simulation example to illustrate the effectiveness and applicability of the proposed method.

Consider an uncertain neutral system with time - varying delays with the following matrices

$A = \begin{bmatrix} -1 & 0 \\ -3 & -2 \end{bmatrix}, A_d = \begin{bmatrix} -0.55 & 0.7 \\ -0.25 & -0.3 \end{bmatrix}, C = \begin{bmatrix} 0.02 & -0.1 \\ -0.1 & 0 \end{bmatrix}$

$B = \begin{bmatrix} 1.4 \\ 0 \end{bmatrix}, D = [0.05 \quad 0], D_d = [0.1 \quad 0], H = 0.3$

The uncertain parameters are given as follows

$$M = \begin{bmatrix} 0.01 & 0 \\ 0 & 0 \end{bmatrix}, N_1 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0 \end{bmatrix}, N_2 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0 \end{bmatrix}, N_3 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0 \end{bmatrix}$$

The disturbance attenuation level is specified to be $\gamma = 0.9$ and we take $h_1 = 1.3,\ h_2 = 2.6,\ \tau_1 = 1.2,\ \tau_2 = 2.9,\ \mu_1 = 0.4,\ \mu_2 = -0.5$. Then by solving the matrix inequality in Theorem 3.2 using **Matlab LMI** toolbox, the gain matrix of the state feed back H_∞ controller can be obtained as

$$K = [-7.9727 \quad -3.7966]$$

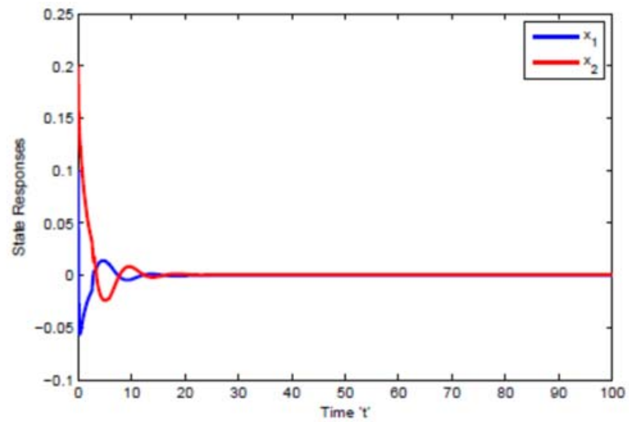

Fig. 1: H_∞ performance of closed-loop system.

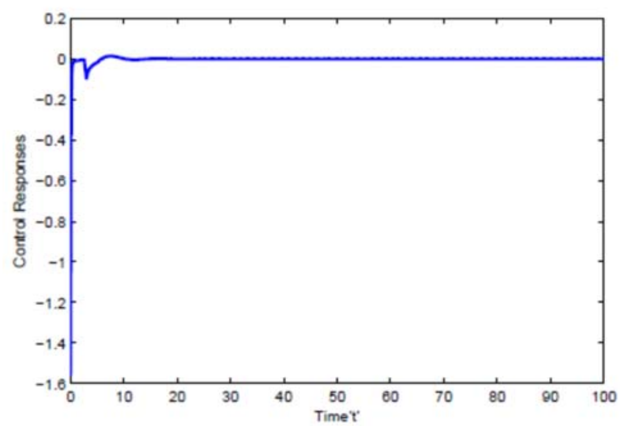

Fig. 2: control response of uncertain system

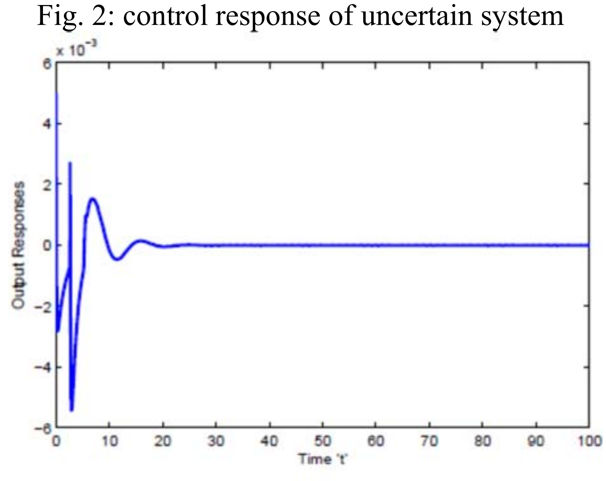

Fig. 3: output response of uncertain system.

Simulation results for state response of neutral system (6) for H_∞ performance is shown in Fig 1. Further, the corresponding controller performance is shown in Fig 2. From the simulation results, it is easy to see that the obtained controller design is suitable to make sure the state trajectories are converging well.

V. CONCLUSION

The problems of stability and dissipative analysis of NCCS have been investigated. In this paper for a time varying random delay technique, some novel Lyapunov-Krasovskii functional candidates were introduced for admissibility and dissipative of NCCS. The derived results are tabulated. At the end, numerical examples were given to demonstrate the modeling and guarantee the effectiveness of the developed approaches.

REFERENCES

1. Du Z., Yue D. and Songlin Hu, H-Infinity stabilization for singular networked cascade control systems with state delay and disturbances, *IEEE Transactions on Industrial Informatics*, 10 (2013) 882-894.
2. Feng Y. and Yagoubi M., On state feedback h-infinity control for discrete-time singular systems, *IEEE Transactions on Automatic Control*, 58 (2013) 2674-2679.
3. Feng Z. and Lam J., Robust reliable dissipative filtering for discrete delay singular systems, *Signal Process*, 92 (2012) 3010-3025.
4. Guo C., Song Q. and Cai W., A neural network assisted cascade control system for air handling unit, *IEEE Transactions on Industrial Electronics*, 54 (2007) 620-628.
5. Han X., Fridman E. and Spurgeon S. K., Sampled-data sliding mode observer for robust fault reconstruction: A time-delay approach, *Journal of the Franklin Institute*, 351 (2014) 2125-2142.
6. Huang C., Bai Y. and Liu X., H-infinity state feedback control for a class of networked cascade control systems with uncertain delay, *IEEE Transactions on Industrial Informatics*,6 (2010) 62-72.
7. Ji X. and Su H., A note on equivalence between two integral inequalities for time-delay systems, *Automatica*, 53 (2015) 244-246.
8. Jiang B., Gao C. and Xie J., Passivity based sliding mode control of uncertain singular Markovian jump systems with time-varying delay and nonlinear perturbations, *Applied Mathematics and Computation*, 271 (2015) 187-200.
9. Jiang S. and Fang H., Robust fault detection for networked control systems with nonlinear disturbances and imperfect measurements, *International Journal of Systems Science*, 44 (2013) 2027-2038.
10. Kao Y. G., Xie J. and Wang C.H., Stabilisation of mode-dependent singular Markovian jump systems with generally uncertain transition rates, *Applied Mathematics and Computation*, 245 (2014) 243-254.
11. Kwon O. M., Park M. J., Lee S. M., Park J. H. and Cha E. J., Stability for neural networks with time varying delays via some new approaches, *IEEE Transactions on neural networks and learning systems*, 24 (2013) 181-193.
12. Lakshmanan S., Park J. H., Lee T. H., Jung H. Y. and Rakkiyappan R., Stability criteria for BAM neural networks with leakage delays and probabilistic time-varying delays, *Applied Mathematics and Computation*, 219 (2013) 9408-9423.
13. Li H., Sun X. and Wu L., State and output feedback control for a class of fuzzy systems with mismatched membership functions, *IEEE Transactions on Fuzzy Systems*, Volume XX number xx (2014).
14. Li S., Yang L., Li K. and Gao Z., Robust sampled-data cruise control scheduling of high speed train, *Transportation Research Part C: Emerging Technologies*, 46 (2014) 274-283.
15. Lin J. and Gao Z., Observers design for switched discrete-time singular time-delay systems with unknown inputs, *Nonlinear Analysis: Hybrid Systems*, 18 (2015) 85-99.
16. Lin J., Shi Y., Fei S. and Gao Z., Reliable dissipative control of discrete-time switched singular systems with mixed time delays and stochastic actuator failures, *IET Control Theory and Applications*, 7 (2013) 1447-1462.
17. Luo M., Zhong S., Wang R. and Kang W., Robust stability analysis for discrete-time stochastic neural networks systems with time-varying delays, *Applied Mathematics and Computation*, 209 (2009) 305-313.
18. Ma Y., Fu L., Jing Y. and Zhang Q., Finite-time H1 control for a class of discrete-time switched singular time-delay systems subject

to actuator saturation, *Applied Mathematics and Computation*, 261 (2015) 264-283.

19. Magdi S., Mahmoud, Abdul-Wahid A. and Saif A., Dissipativity analysis and design for uncertain Markovian jump systems with time-varying delays, *Applied Mathematics and Computation* 219 (2013) 9681-9695.
20. Sakthivel R., Mathiyalagan K. and Marshal Anthoni S., Robust H1 control for uncertain discrete-time stochastic neural networks with time-varying delays, *IET Control Theory Applications*, 6 (2012) 1220-1228.
21. Sakthivel R., Santra S., Mathiyalagan K. and Marshal Anthoni S., Robust reliable sampled-data control for offshore steel jacket platforms with nonlinear perturbations, *Nonlinear Dynamics*, 78 (2014) 1109-1123.
22. Seuret A. and Gouaisbaut F., Wirtinger-based integral inequality: application to time-delay systems, *Automatica*, 49 (2013) 2860-2866.
23. Song Q., Zhao Z. and Liu Y., Stability analysis of complex-valued neural networks with probabilistic time-varying delays, *Neurocomputing*, 159 (2015) 96-104.
24. Teo T., Lakshminarayanan S. and Rangaiah G., Performance assessment of cascade control systems, *Journal of The Institution of Engineers*, Singapore, 45 (2005) 27-38.
25. Wang G., Bo H. and Zhang Q., H1 filtering for time-delayed singular Markovian jump systems with time-varying switching: A quantized method, *Signal Processing*, 109 (2015) 14-24.
26. Wanga J., Park J. H., Shen H. and Wanga J., Delay-dependent robust dissipativity conditions for delayed neural networks with random uncertainties, *Applied Mathematics and Computation*, 221 (2013) 710-719.
27. Weng Y. and Chao Z., Robust Sampled-Data H1 Output Feedback Control of Active Suspension System, *International Journal of Innovative Computing, Information and Control*, 10 (2014) 281-292.
28. Wong H., Zhou B., Lu R. and Xue A., new stability and stabilization criteria for a class of fuzzy singular systems with time varying delay, *Journal of Franklin institute*, 351 (2014) 3766-3781.
29. Wu L., Yang X. and Lam H. K., Dissipativity analysis and synthesis for discrete-Time T-S fuzzy stochastic systems with time-varying delay, *IEEE Transactions on Fuzzy Systems*, 22 (2014) 380-394.
30. Wu L., Zheng W. and Gao H., Dissipativity-based sliding mode control of switched stochastic systems, *IEEE Transactions, Automata Control*, 58 (2013) 785-793.
31. Wu Z., Park J., Su H. and Chu J., Robust dissipativity analysis of neural networks with time-varying delay and randomly occurring uncertainties, *Nonlinear Dynamics*. 74 (2012) 730-734.
32. Wu Z.G., Su H., Shi P. and Chu J., Analysis and synthesis of singular systems with time-Delays, *Computational Intelligence and Complexity (Springer)*, 2013.
33. Xu L., Zhang T. and Yi Y., Robust dissipative control for time-delay stochastic jump systems, *Journal of Systems Engineering and Electronics*, 22 (2011) 314-321.
34. Yang F., Zhang H., Liu Z. and Li R., Delay-dependent resilient-robust stabilisation of uncertain networked control systems with variable sampling intervals, *International Journal of Systems Science*, 45 (2014) 497-508.
35. Yang R., Gao H., Lam J. and Shi P., New stability criteria for neural networks with distributed and probabilistic delays, Circuits Systems. *Signal Process*, 28 (2009) 505-522.
36. Zhang J., Zhang F., Ren M., Hou G. and Fang F., Cascade control of superheated steam temperature with neuro-pid controller, *ISA Transactions*, 51 (2012) 778-785.
37. Zhang T., Comment on Delay-dependent robust H8 filtering for uncertain discrete-time singular systems with interval time-varying delay, *Automatica*, 53 (2015) 291-292.
38. Zhu B., Zhang Q. and Chang C., Delay-dependent dissipative control for a class of non-linear system via Takagi-Sugeno fuzzy descriptor model with time delay, *IET Control Theory and Applications*, 8 (2014) 451-461.
39. Zhu X., Wang Y. and Gan Y., H1 filtering for continuous-time singular systems with time-varying delay, *International Journal of Adaptive Control and Signal Processing*, 49 (2011) 137-154.