FIVE DIMENSIONAL BIANCHI TYPE-III COSMOLOGICAL MODEL WITH NEGATIVE CONSTANT DECELERATION PARAMETER WITH WET DARK FLUID IN BRANS-DICKE THEORY OF GRAVITATION

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Abstract
In the present paper, we have studied the five dimensional Bianchi type-III cosmological model with negative constant deceleration parameter in the scalar tensor theory of gravitation proposed by Brans-Dicke (1961) in the presence of wet dark fluid. Also some Physical properties are studied.
Keywords : Wet dark fluid , negative constant deceleration parameter, Brans-Dicke theory and five dimensional Bianchi type-III space-time, etc.

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§ 1. Introduction
Scalar tensor theories are based on the interaction of scalar field ϕ together with the metric field which form a scalar-tensor field representing gravitational field. There are many alternative theories of gravitation. Among them are scalar tensor theories of gravitation formulated by Brans-Dicke (1961), Nordt-Vedt (1970), Saez-Ballester (1985), which have investigated different aspect of Brans-Dicke (1961) theory. In Brans-Dicke theory the scalar field has inverse dimension of the gravitational constant and its role is confirmed the effect on gravitational equation.

In Brans-Dicke (1961) the field equations for combined scale and tensor field are

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) - \phi^{-1}(\phi_{i;j} - g_{ij}\,\Box\,\phi) \tag{1}$$

and $\Box\,\phi = \phi^{k}_{;k} = 8\pi\phi^{-1}(3+2\omega)^{-1}T$ (2)

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is the Einstein tensor,

T_{ij} is the tensor of matter, ω -is the dimension less coupling constant, comma and semicolon denotes partial and covariant differentiation respectively.

The equation of motion

$$T^{ij}_{;j} = 0 \tag{3}$$

The consequence of the field equation (1) and (2).

The wet dark fluid (WDK) is considered as a model for dark energy. The Bianchi-type-III cosmological model is in the spirit of the generalized Chaplygin gas [Gorini et. al (2004)], where a physically motivated equation of state is offered with properties relevant for the dark energy problem. The motivation stems from an empirical equation of state proposed by Tait (1988) and Hayward (1967) to treat water and aqueous solution.

The equation of state for WDF is very simple and is given by

$$P_{\mathrm{WDF}} = \gamma(\rho_{\mathrm{WDF}} - \rho)$$

where the parameter γ and ρ are consider to be positive i.e $0 \le \gamma \le 1$.

The energy conservation equation is given by

$$\rho_{\mathrm{WDF}} = \frac{\gamma}{1+\gamma}\rho + \frac{c}{v(1+v)}$$

where c is the constant of integration and v is the volume expansion.

we can show that if we take $c > 0$, this fluid will not violate the strong energy condition $P + \rho \ge 0$

$$P_{\mathrm{WDF}} + \rho_{\mathrm{WDF}} = (1+\gamma)\rho - \gamma\rho$$

$$= (1+\gamma)\frac{c}{v^{(1+v)}} \ge 0 .$$

The wet dark fluid has been used as dark energy in the homogeneous, isotropic FRW case by Holman and Naidu (2005). Singh and Chaubey (2008) studied in detail the Bianchi type-I universe with wet dark fluid.

Recently Rehman et. al. (2004) studied some cosmological model in Lyra geometry (1951) using a special law of variation of Hubble's parameter which gives constant deceleration parameter. Reddy et. al. (2001, 2006) have discussed Bianchi-type-I cosmological model with negative constant deceleration parameter in scalar tensor theories formulated by Brans-Dicke (1961) and Saez-Ballister (1985).

In this paper, we have investigated five dimensional Bianchi type-III cosmological model with negative constant deceleration parameter in the scale tensor theory of gravitation proposed by Brans-Dicke(1961) in the presence of wet dark fluid.

The paper is organized as follows: In section -2, we briefly give the five dimensional Bianchi type-III cosmological model and field equations in Brans-Dicke theory of gravitation
Section-3 is the discussion on five dimensional Bianchi type-III cosmological model and field equations in Brans-Dicke theory of gravitation and the last section gives the conclusion.

§ 2. Five dimensional Bianchi type-III cosmological model

We consider Bianchi type-III metric as

$$ds^2 = -dt^2 + A^2(dx^2 + dy^2) + B^2 e^{-2ax} dz^2 + C^2 du^2 \tag{4}$$

where A, B and C are function of time t and a is constant.

The energy momentum tensor is given by

$$T_{ij} = (\rho_{\mathrm{WDF}} + P_{\mathrm{WDF}})u_i u_j - P_{\mathrm{WDF}} \tag{5}$$

where u^i is the flow vector satisfying

$$T_{ij} u^i u^i = 1 . \tag{6}$$

In co-moving co-ordinate system form equation (4) and (6)

$$T_1^1 = T_2^2 = T_3^3 = T_{41}^4 = -P_{\mathrm{WDF}} \quad \text{and} \quad T_5^5 = \rho_{\mathrm{WDF}} . \tag{7}$$

The field equation (1), (2) together with (5-7) yield,

$$\frac{A_{55}}{A} + \frac{B_{55}}{B} + \frac{C_{55}}{C} + \frac{A_5 B_5}{AB} + \frac{A_5 C_5}{AC} + \frac{B_5 C_5}{BC} - \frac{\omega}{2}(\frac{\phi_5}{2})^2 - \frac{\phi_{55}}{\phi} - \frac{\phi_5}{\phi}(\frac{A_5}{A} + \frac{B_5}{B} + \frac{C_5}{C}) = 8\pi\phi^{-1} P_{WDF} , \tag{8}$$

$$\frac{2A_{55}}{A} + \frac{C_{55}}{C} + \frac{2A_5 C_5}{AC} + (\frac{A_5}{A})^2 - \frac{\omega}{2}(\frac{\phi_5}{2})^2 - \frac{\phi_{55}}{\phi} - \frac{\phi_5}{\phi}(\frac{2A_5}{A} + \frac{C_5}{C}) = 8\pi\phi^{-1} P_{WDF} , \tag{9}$$

$$\frac{2A_{55}}{A}+\frac{B_{55}}{B}+\frac{2A_5B_5}{AB}-\frac{a^2}{A^2}+(\frac{A_5}{A})^2-\frac{\omega}{2}(\frac{\phi_5}{2})^2-\frac{\phi_{55}}{\phi}-\frac{\phi_5}{\phi}(\frac{2A_5}{A}+\frac{B_5}{B})=8\pi\phi^{-1}P_{WDF}\,, \tag{10}$$

$$\frac{2A_5B_5}{AB}+\frac{2A_5C_5}{AC}+\frac{B_5C_5}{BC}+(\frac{A_5}{A})^2-\frac{a^2}{A^2}+\frac{\omega}{2}(\frac{\phi_5}{2})^2-\frac{\phi_5}{\phi}(\frac{2A_5}{A}+\frac{B_5}{B}+\frac{C_5}{C})=-8\pi\phi^{-1}\rho_{WDF}\,, \tag{11}$$

$$\frac{A_5}{A}-\frac{B_5}{B}=0\,. \tag{12}$$

$$\phi^{i}_{;j}=\phi_{55}+\phi_5(\frac{2A_5}{A}+\frac{B_5}{B}+\frac{C_5}{C})=\frac{8\pi\phi^{-1}}{(3+2\omega)}(\rho_{WDF}-3P_{WDF})\,. \tag{13}$$

Here suffix 5 represent the time derivative.
The equation (3), which is the consequence of the field equation take the form

$$(\rho_{WDF})_5+(\rho_{WDF}+P_{WDF})(\frac{2A_5}{A}+\frac{B_5}{B}+\frac{C_5}{C})=0\,. \tag{14}$$

From equation (12), we get

$$A=B\,. \tag{15}$$

Using equation (15), the set of equation (8-14) written as,

$$\frac{2B_{55}}{B}+\frac{C_{55}}{C}+(\frac{B_5}{B})^2+\frac{2B_5C_5}{BC}-\frac{\omega}{2}(\frac{\phi_5}{2})^2-\frac{\phi_{55}}{\phi}-\frac{\phi_5}{\phi}(\frac{2B_5}{B}+\frac{C_5}{C})=8\pi\phi^{-1}P_{WDF}\,, \tag{16}$$

$$\frac{2B_{55}}{B}+\frac{C_{55}}{C}+(\frac{B_5}{B})^2+\frac{2B_5C_5}{BC}-\frac{\omega}{2}(\frac{\phi_5}{2})^2-\frac{\phi_{55}}{\phi}-\frac{\phi_5}{\phi}(\frac{2B_5}{B}+\frac{C_5}{C})=8\pi\phi^{-1}P_{WDF}\,, \tag{17}$$

$$\frac{3B_{55}}{B}+2(\frac{B_5}{B})^2-\frac{a^2}{B^2}+(\frac{B_5}{B})^2-\frac{\omega}{2}(\frac{\phi_5}{2})^2-\frac{\phi_{55}}{\phi}-\frac{\phi_5}{\phi}(\frac{3B_5}{B})=8\pi\phi^{-1}P_{WDF}\,, \tag{18}$$

$$3(\frac{B_5}{B})^2+\frac{3B_5C_5}{BC}-\frac{a^2}{B^2}+\frac{\omega}{2}(\frac{\phi_5}{2})^2-\frac{\phi_5}{\phi}(\frac{3B_5}{B}+\frac{C_5}{C})=-8\pi\phi^{-1}\rho_{WDF}\,, \tag{19}$$

$$\phi_{55}+\phi_5(\frac{3B_5}{B}+\frac{C_5}{C})=\frac{8\pi\phi^{-1}}{(3+2\omega)}(\rho_{WDF}-3P_{WDF})\,, \tag{20}$$

$$(\rho_{WDF})_5+(\rho_{WDF}+P_{WDF})(\frac{3B_5}{B}+\frac{C_5}{C})=0\,. \tag{21}$$

There are five unknown variable B, C, ρ_{WDF}, P_{WDF} and ϕ. Therefore, to obtain the solution one extra condition is essential, we consider the equation of state in the form

$$\rho_{WDF}=3P_{WDF}\,. \tag{22}$$

This shows that matter distribution with disordered radiation is analogous to $\rho=3P$ in general relativity.

The equations (16-21) are highly nonlinear. Therefore, we assume a relation between metric coefficients given by

$$B = \mu C. \tag{23}$$

where μ is constant.
Using (22), the set of equations (16-21) written as:

$$\frac{2B_{55}}{B} + \frac{C_{55}}{C} + \left(\frac{B_5}{B}\right)^2 + \frac{2B_5C_5}{BC} - \frac{\omega}{2}\left(\frac{\phi_5}{2}\right)^2 - \frac{\phi_5}{\phi}\left(\frac{2B_5}{B} + \frac{C_5}{C}\right) = 8\pi\phi^{-1}P_{WDF}, \tag{24}$$

$$\frac{2B_{55}}{B} + \frac{C_{55}}{C} + \left(\frac{B_5}{B}\right)^2 + \frac{2B_5C_5}{BC} - \frac{\omega}{2}\left(\frac{\phi_5}{2}\right)^2 - \frac{\phi_5}{\phi}\left(\frac{2B_5}{B} + \frac{C_5}{C}\right) = 8\pi\phi^{-1}P_{WDF}, \tag{25}$$

$$\frac{3B_{55}}{B} + 2\left(\frac{B_5}{B}\right)^2 - \frac{a^2}{B^2} + \left(\frac{B_5}{B}\right)^2 - \frac{\omega}{2}\left(\frac{\phi_5}{2}\right)^2 - \frac{\phi_5}{\phi}\left(\frac{3B_5}{B}\right) = 8\pi\phi^{-1}P_{WDF}, \tag{26}$$

$$3\left(\frac{B_5}{B}\right)^2 + \frac{3B_5C_5}{BC} - \frac{a^2}{B^2} + \frac{\omega}{2}\left(\frac{\phi_5}{2}\right)^2 - \frac{\phi_5}{\phi}\left(\frac{3B_5}{B} + \frac{C_5}{C}\right) = -8\pi\phi^{-1}\rho_{WDF}, \tag{27}$$

$$\phi_{55} + \phi_5\left(\frac{3B_5}{B} + \frac{C_5}{C}\right) = 0, \tag{28}$$

$$(\rho_{WDF})_5 + \frac{4}{3}(\rho_{WDF} + P_{WDF})\left(\frac{3B_5}{B} + \frac{C_5}{C}\right) = 0. \tag{29}$$

The above sets of equations are solved, using special law of variation of Hubble's parameter proposed by Berman (1983), which gives constant deceleration parameter of the model.

The deceleration parameter is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{aa_{55}}{(a_5)^2} = \text{constant} \tag{30}$$

where $a = (B^3Ce^{-ax})^{1/4}$ is scale factor and the constant is taken as negative.

The solution of equation (30) is given by

$$a = (\alpha t + \beta)^{\frac{1}{1+q}} \tag{31}$$

where $\alpha \neq 0$, $\beta \neq 0$ are constant of integration and $1 + q > 0$.

Solving above equation, we get

$$A = B = \mu^{1/4}e^{ax/4}(\alpha t + \beta)^{\frac{1}{1+q}}, \tag{32}$$

$$C = \mu^{-3/4}e^{ax/4}(\alpha t + \beta)^{\frac{1}{1+q}}. \tag{33}$$

Using (32) and (33), the metric (4) can be written as

$$ds^2 = -dt^2 + \mu^{1/2}e^{ax/2}(\alpha t+\beta)^{\frac{2}{1+q}}(dx^2+dy^2) + \mu^{1/2}e^{ax/2}(\alpha t+\beta)^{\frac{2}{1+q}}dz^2 + \mu^{-3/2}e^{ax/2}(\alpha t+\beta)^{\frac{2}{1+q}}du^2$$
(34)

The proper choice of coordinates and constants of integrations the equation (34) reduce to

$$ds^2 = -dT^2 + \mu^{1/2}e^{ax/2}T^{\frac{2}{1+q}}(dX^2+dY^2) + \mu^{1/2}e^{ax/2}T^{\frac{2}{1+q}}dZ^2 + \mu^{-3/2}e^{ax/2}T^{\frac{2}{1+q}}dU^2 . \quad (35)$$

§ 3. Some Physical properties

Here, we considered the five dimensional Bianchi-type-III cosmological models with negative constant of deceleration parameter in the frame work of Brans-Dicke (1961) scalar tensor theory of gravitation. The expression for the scale expansion scalar θ, shear scale σ^2, Hubble parameter and volume of the mode (35) are as follows

$$V = \sqrt{-g} = T^{\frac{4}{1+q}} , \quad (36)$$

$$\theta = \frac{\alpha}{(1+q)T} , \quad (37)$$

$$\sigma^2 = \frac{\alpha^2}{8(1+q)^2 T^2} , \quad (38)$$

and $$H = \left(\frac{\alpha}{1+q}\right)\frac{1}{T} . \quad (39)$$

we observed that all the physical quantities at initial point $T = 0$, diverges.

But $T \to \infty$ then volume become infinitely large and expression tensor, shear scale and Hubble parameter tends to Zero.

$$\therefore \lim_{T\to\infty} (\sigma/\theta)^2 \neq 0 .$$

Then the model does not approach isotropy for large value of T .

The expression for Brans-Dicke scalar field ϕ, the density and pressure are given by

$$\phi = k_1 e^{-ax}\left(\frac{q+1}{q-3}\right)T^{\frac{q-3}{q+1}} + \phi_0 , \quad (40)$$

$$\rho_{WDF} = k_2 (e^{-4ax}/3)T^{\frac{-16}{3(q+1)}} , \quad (41)$$

$$P_{WDF} = \frac{k_2}{3}(e^{-4ax}/3)T^{\frac{-16}{3(q+1)}} . \quad (42)$$

Thus from equation (36)-(42), we observed that at initial point $T = 0$, then the physical quantities ρ and P diverge but the scale field ϕ has no initial singularity. Hence the universe starts with an infinite rate of expansion and measure of anisotropy. As $T \to \infty$ then density and pressure tends to zero.

Conclusion:

We have obtained the five dimensional Bianchi-type-III cosmological model in Brans-Dicke (1961) scalar tensor theory of gravitation in presence of wet dark fluid using special law of variation for Hubble parameter proposed by Bermann (1983). The model represents a radiating universe in Brans-Dicke (1961) theory of gravitation.

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