EXISTENCE OF $[z-\frac{1}{\sqrt{2}}(t_1+t_2)]$-TYPE AND $[(t_1+t_2)/z\sqrt{2}]$-TYPE PLANE WAVES IN V_6 FOR BIMETRIC RELATIVITY

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Abstract

The bimetric relativity admits $[z-\frac{1}{\sqrt{2}}(t_1+t_2)]$-type and $[(t_1+t_2)/z\sqrt{2}]$-type plane gravitational waves in six dimensional space-times V_6 having two time axes where in the later case the space-times can be reduced to conformal one.

§1.Introduction

Reformulating Karade's (1994) definition of plane wave, we have obtained the plane wave solutions of the field equations of the field equations $N_i^j = 0$ and established the existence of $[z-\frac{1}{\sqrt{2}}(t_1+t_2)]$-type and $[(t_1+t_2)/z\sqrt{2}]$-type plane gravitational waves in four-dimensional space-times V_4 having two time axes with reference to the papers [1] and [2] respectively. Extending this work to higher five dimensional space-times V_5 having two time axes, we have studied these two types of waves in the papers refer them to [3] and [4]. It has been observed that the expressions for various quantities obtained in V_4 are retained their forms in V_5 too. Furthermore in the paper refer it to [5], we have obtained plane wave solutions in six dimensional space-times V_6 having two time axes are given by g_{ij} which satisfied

$$Q\rho_i^j + P\sigma_i^j = 0 \qquad (1.1)$$

which further breaks in

$$\overline{\overline{w}}_4\rho_i^j + \overline{w}_4\sigma_i^j = 0 = \overline{\overline{\phi}}_4\rho_i^j + \overline{\phi}_4\sigma_i^j, \quad \overline{\overline{w}}_5\rho_i^j + \overline{w}_5\sigma_i^j = 0 = \overline{\overline{\phi}}_5\rho_i^j + \overline{\phi}_5\sigma_i^j, \qquad (1.2)$$

where $w_4 = t_2 + \phi_4 z$, $\quad w_5 = t_2 + \phi_5 z$, $\quad \phi_4 = \frac{Z_{,4}}{Z_{,6}}$, $\quad \phi_5 = \frac{Z_{,5}}{Z_{,6}}$,

$$M_4 = \overline{w}_4 - \overline{\phi}_4 z, \qquad M_5 = \overline{w}_5 - \overline{\phi}_5 t_1,$$

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$$\rho_i^j = [(\phi_4^2 - \phi_5^2) - 1]g^{hj}\overline{g}_{hi} \quad \text{and} \quad \sigma_i^j = \frac{d}{dZ}\{[1-(\phi_4^2-\phi_5^2)]g^{hj}\overline{g}_{hi}\}.$$

In the present paper, we study the solutions (1.1) for $[z-\frac{1}{\sqrt{2}}(t_1+t_2)]$-type and $[(t_1+t_2)/z\sqrt{2}]$-type plane gravitational waves in six dimensional space-times V_6 having two time axes for BR theory of Rosen (1973,74) where at each point of the space-time there are two line elements

$$ds^2 = g_{ij}dx^i dx^j \qquad \text{and} \qquad d\sigma^2 = f_{ij}dx^i dx^j \tag{1.3}$$

§ 2. $[z - \frac{1}{\sqrt{2}}(t_1 + t_2)]$-type plane wave in V_6

Let $Z = [z - \frac{1}{\sqrt{2}}(t_1 + t_2)] \quad \Rightarrow Z_{,4} = 1, \quad Z_5 = -\frac{1}{\sqrt{2}}, \quad Z_{,6} = -\frac{1}{\sqrt{2}}.$

Then $\phi_4 = \frac{Z,_4}{Z,_6} = -\sqrt{2}, \qquad \phi_5 = \frac{Z,_5}{Z,_6} = 1.$

Also $w_4 = t_2 + \phi_4 z = -Z\sqrt{2} - t_1, \qquad w_5 = t_2 + \phi_5 t_1 = -Z\sqrt{2} + z\sqrt{2}$

$\Rightarrow \overline{w}_4 = -\sqrt{2}, \qquad \overline{w}_5 = -\sqrt{2}.$

Hence $M_4 = \overline{w}_4 - \overline{\phi}_4 z = -\sqrt{2}, \qquad M_5 = \overline{w}_5 - \overline{\phi}_5 t_1 = -\sqrt{2}$

$\Rightarrow P = -\sqrt{2} \qquad \because M_4 = M_5 = P$

and $N_4 = \overline{\overline{w}}_4 - \overline{\overline{\phi}}_4 z = 0, \qquad N_5 = \overline{\overline{w}}_5 - \overline{\overline{\phi}}_5 t_1 = 0$

$\Rightarrow Q = 0 \qquad \because N_4 = N_5 = Q$

$$\Rightarrow \sigma_i^j = 0. \tag{2.1}$$

With the above values, the L.H.S. of the field equations (1.2) become zero and hence the equation is identically satisfied. Therefore, it implies that $[z - \frac{1}{\sqrt{2}}(t_1 + t_2)]$-type plane gravitational wave in six dimensional space-time V_6 having two time axes exists in biometric relativity.

§ 3. $[(t_1 + t_2)/z\sqrt{2}]$-type plane wave in V_6

Let $Z = [(t_1 + t_2)/z\sqrt{2}]$

$\Rightarrow Z_{,4} = -(t_1 + t_2)/z^2\sqrt{2}, \qquad Z_{,5} = \frac{1}{z\sqrt{2}}, \qquad Z_{,6} = \frac{1}{z\sqrt{2}}$

Then $\phi_4 = \frac{Z,_4}{Z,_6} = -Z\sqrt{2}, \qquad \phi_5 = \frac{Z,_5}{Z,_6} = 1.$

Also $w_4 = t_2 + \phi_4 z = -t_1, \qquad w_5 = t_3 + \phi_5 t_1 = zZ\sqrt{2},$

$\Rightarrow \overline{w}_4 = 0, \qquad \overline{w}_5 = z\sqrt{2}.$

Hence $M_4 = \overline{w}_4 - \overline{\phi}_4 z = z\sqrt{2}, \qquad M_5 = \overline{w}_5 - \overline{\phi}_5 t_1 = z\sqrt{2}$

$\Rightarrow P = z\sqrt{2}, \qquad \because M_4 = M_5 = P$

and $N_4 = \overline{\overline{w}}_4 - \overline{\overline{\phi}}_4 z = 0, \qquad N_5 = \overline{\overline{w}}_5 - \overline{\overline{\phi}}_5 t_1 = 0$

$\Rightarrow Q = 0 \qquad \because N_4 = N_5 = Q$

$$\Rightarrow \sigma_i^j = 0. \tag{3.1}$$

and the field equation (1.2) reduces to

$$\{[1 - (\phi_4^2 - \phi_5^2)]g^{hj}\overline{g}_{hi} = c_i^j \qquad \text{i.e.,} \qquad 2[1 - Z^2]g^{hj}\overline{g}_{hi} = c_i^j$$

where c_i^j are constants.

If we choose δ_i^j in particular, we get

$$[1-Z^2]g^{hj}\overline{g}_{hi} = \delta_i^j \qquad \text{i.e., } [1-Z^2]\overline{g}_{ki} = g_{ki}$$

and then $g_{ki} = D_{ki}[\frac{1+Z}{1-Z}]^{1/2}$

where D_{ki} are constants.

Noting (1.3), the space-times V_6 admitting $[(t_1+t_2)/z\sqrt{2}]$-type plane gravitational waves becomes

$$ds^2 = \left[\sqrt{\frac{z\sqrt{2}+(t_1+t_2)}{z\sqrt{2}-(t_1+t_2)}}\right] D_{ij}dx^i dx^j$$

which is reducible to a conformal space-times.

Conclusion

The biometric relativity admits $[z-\frac{1}{\sqrt{2}}(t_1+t_2)]$-type and $[(t_1+t_2)/z\sqrt{2}]$ -type plane gravitational waves in six dimensional space-times V_6 having two time axes where in the later case the space-times can be reduced to conformal one.

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